# KEY TO GREENLEAF'S INTELLECTUAL ARITHMETIC

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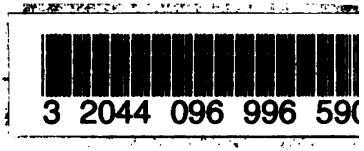
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# KEY

TO THE

# INTELLECTUAL ARITHMETIC,

CONTAINING

ANSWERS AND SOLUTIONS OF THE MORE DIFFICULT EXAMPLES IN THAT WORK.

FOR THE USE OF TEACHERS.

By BENJAMIN GREENLEAF, A.M.,
AUTHOR OF A SERIES OF ARITHMETICS, ETC.

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# PREFACE.

SINCE the publication of the Intellectual Arithmetic, it has been found expedient to prepare a Key, to lighten the labor of teachers. It would, however, be impracticable and unnecessary to furnish a full analysis of every example. The solutions which involve new principles are given in full, unless those furnished in the Arithmetic itself form a sufficient guide; if other examples require any attention, either the answers alone are given, or the leading points of the analysis are briefly expressed, using signs whenever it is practicable, and contracting more than would be allowable in recitation, even by the most advanced pupils.

It will be observed, that, in using the sign $\times$, the multiplier is sometimes placed before, and sometimes after it.

As this work is intended for the use of *teachers* only, language is sometimes used in the discussion of principles which young pupils might not understand, and occasionally more than one mode of analysis is given, leaving the teacher to select the one which will be most consistent with his own method of teaching.

Lessons LII. – LXIII. were prepared for the convenience of those teachers who wish to train their pupils to perform mental operations on large numbers. Others can omit any portion of these lessons if they choose to do so, as such a course would not mar the unity of the work.

MAY 15, 1858.

CAMBRIDGE:
*ELECTROTYPED* AND PRINTED BY METCALF AND COMPANY.

# KEY

TO

# GREENLEAF'S

# INTELLECTUAL ARITHMETIC.

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LESSONS I.–X. treat of Addition and Subtraction, and require no explanation here.

LESSON XI. PAGE 24, NOTE. This formula may be introduced by either *since*, *as*, *if*, or a similar conjunction, or otherwise varied to suit the taste of different teachers. The reasoning, fully expressed, would include the fact that 2 apples cost twice as much as one apple. The pupil should understand that the suppressed proposition is always understood; but it is so intimately connected with the very idea of number, and so readily supplied, that it may not be necessary always to express it, any more than it is always necessary to express both the premises of every syllogism. A formula which is used so constantly, should be as brief as possible, provided it be at the same time accurate.

## LESSON XIII.—PAGES 28–31.

10. If one travels 3 and the other 4 miles an hour, at the end of one hour they will be $3+4=7$ miles apart; in 5 hours they will be 5 times 7 miles, which are 35 miles, apart.

19. If one travels 4 and the other 3 miles an hour, in one hour they will be $4+3=7$ miles nearer each other; in 5 hours they will be 5 times 7 miles, or 35 miles, nearer each other. They were 50 miles apart when they started, hence the distance between them will be the difference between 50 *and 35 miles*, which is 15 miles.

37. $175. [$25 \times 6 + 25$]
38. 96 feet. [$(12 - 4) \times 12$]
45. 20 quarts. [$64 - (24 + 20)$]
46. 80 chickens. [$(3 + 15 + 22) \times 2$]
47. 30 nuts. [$75 - (20 + 25)$]
48. Each obtained 52 cents.

## LESSON XVIII. — Pages 42-44.

16. 63 cents are 7 times 9 cents; therefore, for 63 cents there can be bought 7 times 4 oranges. Or it might be stated thus: — As many times 4 oranges can be bought for 63 cents, as 9 cents is contained times in 63 cents; 9 is contained in 63, 7 times; therefore there can be bought 7 times 4 oranges, which are 28 oranges.

17. $28. [1 ton costs $7, and 4 tons cost 4 times $7.]

18. 3 cents. [$(60 \div 4) \times 3 = 45$; $(60 \div 5) \times 4 = 48$; $48 - 45 = 3$.]

26. If it take 4 men 8 days, it will take 1 man 4 times 8 days, which are 32 days; if it take 1 man 32 days, it will take 16 men one sixteenth of 32 days, which is 2 days.

27. If it require 16 men to do it in 2 days, it will require 2 times 16 men, or 32 men, to do it in one day, and it will require one eighth of 32 men to do it in 8 days. Or,

If it take 16 men 2 days, it will take 1 man 16 times 2 days, or 32 days, and if it take 1 man 32 days, it will take as many men to do it in 8 days as 8 days is contained times in 32 days.* Ans. 4 men.

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* It will be observed that, in dividing 32 by 8, we use different expressions in the two solutions here given. Where the numbers are of different denominations, and the dividend is of the same kind as the result, we may use the expression, "one eighth of 32 men"; but where the denominations are the same, we must say, "as many as 8 days is contained times in 32 days." In the latter case the divisor and dividend are both denominate and the quotient abstract; but the former is really multiplying by a fraction, the multiplicand and *product (or dividend* and quotient) being denominate, and the *multiplier (or divisor) abstract.* Several examples in Lessons XIV., XVI.,

29. 6 days. [$(9 \times 8) \div 12$]

31. 15 hours. [$30 \div (7 - 5)$]

32. 15 minutes. [$(40 \div 4) - (40 \div 5) = 2$; $30 \div 2 = 15$]

33. $32. [$(12 + 20 + 64) \div 8$]

34. $5. [$(18 \div 3) \times 10 \div 12$]

35. 9 bushels. [$27 \times 8 \div 9$]

36. If 1 bushel of corn is worth 2 bushels of oats, 2 bushels of corn are worth 2 times 2 bushels of oats, which are 4 bushels of oats; hence 4 bushels of oats are worth 1 bushel of wheat, and 20 bushels of oats are worth as many bushels of wheat as 4 is contained times in 20. Ans. 5 bushels.

37. 3 days. | 38. 9 pounds. [$81 \div (3 \times 3)$]

39. If 10 gallons run in and 5 gallons run out in an hour, 5 gallons remain in the cistern, and it will take as many hours to fill it as 5 gallons is contained times in 60 gallons.

Ans. 12 hours.

## LESSON XIX. — Pages 45, 46.

### United States Money.

1. Since in 1 cent there are 10 mills, in 2 cents there will be 2 times 10 mills, which are 20 mills. Or,

There are 2 times as many mills in 2 cents as in 1 cent; in 1 cent there are 10 mills, and in 2 cents there will be 2 times 10 mills, which are 20 mills. Therefore, in 2 cents there are 20 mills.

All examples in Reduction Descending may be explained in a manner similar to one of the above forms.

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and XVII. might be solved by the former method, as well as the one given in Lesson XIV.

The 26th question might also be solved by finding how many men it would take to do it in one day. All similar examples may be explained in these two ways; but perhaps it is as well to select the method which makes the intermediate results of the same denomination as the answer is to be. The facts that it will take 16 men $\frac{1}{2}$ as long as it takes 4 men, and will require $\frac{1}{4}$ as many men to do the work in 8 days as to do it in 2 days, give a third solution of questions 26 and 27.

2. As there is 1 cent in 10 mills, there will be as many cents in 20 mills as 10 mills is contained times in 20 mills, which are 2. Ans. 2 cents.

All examples in Reduction Ascending may be explained in a similar manner.

In 47 mills there are 4 cents, and 7 mills remaining.

4. In 43 cents there are 4 dimes, and 3 cents remaining.

10. If 1 yard costs 5 mills, 20 yards will cost 20 times 5 mills, which are 100 mills. As there is 1 cent in 10 mills, there will be as many cents in 100 mills as 10 mills is contained times in 100 mills, which are 10. Ans. 10 cents.

11. \$ 6. $[(2 \times 30) \div 10]$

12. 25 lbs. $[(10 \times 5) \div 2]$

13. If 7 lbs. cost 70 cents, 1 pound will cost one seventh of 70 cents, which is 10 cents. There are 10 dollars in 1 eagle; in 1 dollar there are 100 cents, and in 10 dollars there will be 10 times 100 cents, which are 1,000 cents. If 1 pound costs 10 cents, we can buy as many pounds for 1,000 cents as 10 cents is contained times in 1,000 cents, which are 100. Ans. 100 lbs.

### English Money.

8. Since in 1 pound there are 20 shillings, in 2 pounds there will be 2 times 20 shillings, which are 40 shillings, and 6 shillings added give 46 shillings.

9. 6 shillings. $[(3 \times 24) \div 12]$

10. 8£. $[(5 \times 32) \div 20]$

## LESSON XX.—Pages 46-48.

### Troy Weight.

9. \$ 5.40. $[6 \times (20 \times 4 + 10)]$

10. 405 dimes. $[9 \times (20 \times 2 + 5)]$

### Avoirdupois Weight.

*9. \$ 240.* $[20 \times 6 \times 2]$

10. 163 lbs. [25 × 6 + 13]
11. $22.50. [(4 × 2 + 1) × 25 × 10]
12. $180. [20 × 3 × (15 ÷ 5)]

## LESSON XXI. — Pages 48–50.

### Long Measure.

12. 96 fur.; 144 fur.
13. 40 m.; 120 m.; 240 m.
14. 102 in.; 65 in.

### Cloth Measure.

7. $80.
8. 61 nails.
9. 23 quarters.
10. 5 yd., and 3 qr. rem.
11. $1.80.
12. $4.

## LESSON XXII. — Pages 50, 51.

### Square Measure.

8. 108 sq. rods. | 9. $800. | 10. 260 sq. rods.

### Solid Measure.

8. $30. [3 × 2 × 5]
9. $25. [(40 × 2 + 20) ÷ 4]

## LESSON XXIII. — Pages 52, 53.

### Liquid Measure.

8. $1.60; $2.56; $3.20.
9. 24 cts.; 12 cts.: 3 cts.
10. $1.28.
11. $3.
12. 20 gills.
13. $42. [3 × (70 ÷ 5)]

### Dry Measure.

7. 96 cts.
8. $1.28.
9. *64 qts.*; 96 qts.
10. 184 pints.
11. $3.20.

## LESSON XXIV.—Pages 53–55.

### Table of Time.

9. 2 hours. | 10. 5 months. | 11. 5 months.
12. 4 days. [6 pairs in one day; $24 \div 6 = 4$]

### Miscellaneous Table.

3. $14.40.
5. $24.
6. 60 cts.
7. $2.
8. 16 cts. [$2 \times (24 \div 3)$]
9. $22.
10. $6.
11. 20 cts.
12. $6.

## LESSON XXV.—Pages 56–58.

10. Since there is one whole one in 2 halves, there will be as many whole ones in 6 halves as 2 halves is contained times in 6 halves, which are 3.

12. $\frac{1}{2}$ of 1 is $\frac{1}{2}$, and $\frac{1}{2}$ of 5 is 5 times $\frac{1}{2}$, which are $\frac{5}{2}$. As in $\frac{2}{2}$ there is one whole one, in $\frac{5}{2}$ there will be as many whole ones as $\frac{2}{2}$ is contained times in $\frac{5}{2}$, which are 2 and 1 half.

13. 7 is as many times 2 as 2 is contained in 7, which are 3 times 2 and $\frac{1}{2}$ of 2.

28. 15 times 3 and $\frac{2}{3}$ of 3.

32. If 5 quarts cost 35 cents, 1 quart will cost $\frac{1}{5}$ of 35 cents, which is 7 cents; 7 cents is contained in 21 cents 3 times; hence 1 quart will cost $\frac{1}{3}$ of 21 cents. If 1 quart costs $\frac{1}{3}$ of 21 cents, 2 quarts will cost 2 times $\frac{1}{3}$, which are $\frac{2}{3}$, of 21 cents. Or,

Since 1 quart costs 7 cents, it will cost $\frac{7}{21}$ of 21 cents, and 2 quarts will cost 2 times $\frac{7}{21}$, which are $\frac{14}{21}$. [The reasoning is more fully given in the 37th.]

37. 1 is $\frac{1}{4}$ of 4, since 1 taken 4 times $= 4$; 2 is 2 times $\frac{1}{4} = \frac{2}{4}$ of 4; and 3 is $\frac{3}{4}$ of 4.

46. $\frac{1}{2}$, or $\frac{4}{16}$, of $16. The pupil is not supposed to understand *reducing fractions* to their lowest terms, and hence he

must use one of the two methods by which the 32d example is explained.

48. $\frac{1}{4}$ of \$ 160 is \$ 40, and $\frac{3}{4}$ will be 3 times \$ 40, which are \$ 120, and \$ 5 added make \$ 125, the cost of the wagon.

## LESSON XXVI.—Pages 58–60.

1. It will be the same part of a barrel that \$ 2 is of \$ 10. \$ 2 is contained in \$ 10, 5 times, and is $\frac{1}{5}$ of \$ 10; hence \$ 2 will buy $\frac{1}{5}$ of a barrel. Or,

\$ 2 is $\frac{2}{10}$ of \$ 10; hence \$ 2 will buy $\frac{2}{10}$ of a barrel.

4. 1 is $\frac{1}{5}$ of 5, since 1 taken 5 times = 5, and 3 will be 3 times $\frac{1}{5}$ of 5, which are $\frac{3}{5}$ of 5. The pupil will soon be able to answer questions of this kind without going through this process of reasoning each time.

5. Once 5 and $\frac{3}{5}$ of 5.

9. $\frac{1}{5}$ of it is worth $\frac{1}{5}$ of \$ 75, which is \$ 15. \$ 5 is contained in \$ 15, 3 times.

11. We first find the cost of 1 pound, and then of 2 pounds, which is 14 cents. 14 cents is $\frac{1}{3}$, or $\frac{14}{42}$, of 42 cents. [See 46th example, last Lesson.]

24. The cost was $\frac{6}{7}$ of itself, and $\frac{1}{7}$ added makes $\frac{7}{7}$. $\frac{1}{7}$ of \$ 120 is \$ 20, and $\frac{7}{7}$ are 7 times \$ 20, which are \$ 140.

39. If 4 questions are $\frac{4}{7}$ of the number, $\frac{1}{7}$ will be $\frac{1}{4}$ of 4 questions, which is 2 questions, and $\frac{7}{7}$, or the whole number, will be 7 times 2 questions, which are 14 questions. Or,

If 4 questions are $\frac{4}{7}$ of the number, $\frac{1}{4}$ of 4 questions, or 2 questions, will be $\frac{1}{7}$; if 2 questions is $\frac{1}{7}$, $\frac{7}{7}$, or the whole number, will be 7 times 2 questions, which are 14 questions.

| | | |
|---|---|---|
| 47. 30 walnuts. | 49. 75 cents. | 51. \$ 6. |
| 48. 42 dimes. | 50. \$ 5.60. | 52. \$ 13. |

## LESSON XXVII.—Pages 61, 62.

1. If 8 is $\frac{1}{4}$ of the number, $\frac{4}{4}$, or the whole number, will be 4 times 8, which are 32. Or,

*8 is $\frac{1}{4}$ of* 4 times 8, which are 32.

8. 8. [Analyzed like the 39th, last Lesson.]

18. We find, as before, that 14 is $\frac{7}{8}$ of 16. 16 is 4 times 4.

28. We find, as in the 1st example, that 15 is $\frac{1}{4}$ of 60. 60 is 5 times $\frac{1}{5}$ of 60, which is 12.

36. $\frac{1}{8}$ of 32 is 4, and $\frac{5}{8}$ will be 5 times 4, which are 20; 20 is 4 times 5.

43. 4 times 3 are 12; $\frac{1}{3}$ of 3 is 1, and $\frac{2}{3}$ will be 2 times 1, which are 2; 2 added to 12 make 14.

51. As before, we find that 8 times 10 and $\frac{9}{10}$ of 10 are 89. 89 is 17 times 5 and $\frac{4}{5}$ of 5.

53. 9 times 12 are 108; $\frac{1}{12}$ of 12 is 1, and $\frac{11}{12}$ will be 11 times 1, which are 11; 108 less 11 is 97; 97 is 9 times 10 and $\frac{7}{10}$ of 10.

58. $\frac{1}{9}$ of 36 is 4, and $\frac{5}{9}$ will be 5 times 4, which are 20; if 20 is $\frac{2}{3}$ of some number, $\frac{1}{6}$ of that number will be $\frac{1}{3}$ of 20, which is 10, and $\frac{6}{6}$, or the whole number, will be 5 times 10, which are 50; 50 is 5 times 10.

62. We find that $\frac{5}{12}$ of 60 is 25, and $\frac{1}{5}$ of 30 is 5; 25 is 5 times 5.

## LESSON XXVIII. — PAGES 63 – 66.

6. Since 1 pound costs 16 cents, 3 pounds will cost *3 times* 16 cents, which are 48 cents; $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of 16 cents, $\frac{1}{4}$ of 16 cents is 4 cents, and $\frac{3}{4}$ will be 3 times 4 cents, which are 12 cents; 12 cents added to 48 cents make 60 cents. Or,

Since 1 pound costs 16 cents, 3 pounds and $\frac{3}{4}$ of a pound will cost 3 times 16 cents, and $\frac{3}{4}$ of 16 cents. [Explain the remainder like the 43d in the last Lesson, except that we have 16 cents, instead of an abstract number.]

19. Explain like the 28th in the last Lesson; or as follows: $\frac{1}{2}$ of 4 times is 2 times, hence \$ 5 is twice the price of the hat; \$ 5 is twice $\frac{1}{2}$ of 5, which is \$ $2\frac{1}{2}$.

21. 5 cents. [Analyzed like the 19th.]

*22. $\frac{1}{6}$ of 12 years* is 2 years, and $\frac{7}{6}$ will be 7 times 2 years, *which are 14 years*; if 14 years are $\frac{7}{8}$ of her sister's age, $\frac{1}{8}$ will

be $\frac{1}{7}$ of 14 years, which is 2 years, and $\frac{6}{6}$, or her sister's age, will be 8 times 2 years, which are 16 years. Or,

If $\frac{7}{8}$ of Emma's age is $\frac{7}{8}$ of her sister's age, $\frac{1}{8}$ of Emma's age will be $\frac{1}{7}$ of $\frac{7}{8}$, or $\frac{1}{8}$, of her sister's age; $\frac{1}{8}$ of 12 years is 2 years, and if 2 years is $\frac{1}{8}$ of her sister's age, $\frac{8}{8}$ will be 8 times 2 years, which are 16 years.

23. We find $\frac{5}{8}$ to be 25 feet, and 5 feet added make 30 feet.

25. $\frac{4}{5}$ of \$ 40 are \$ 16, and \$ 5 added make \$ 21, the cost of the chain. \$ 40 + \$ 21 = \$ 61.

28. Each boy would have $\frac{1}{4}$ of $\frac{1}{3}$ of it, which is $\frac{1}{3}$ of it; $\frac{1}{3}$ of 50 cents is 10 cents; the one who could tell would have $\frac{1}{2}$ of 10 cents, or 5 cents, in addition; hence he would receive 10 cents + 5 cents = 15 cents.

33. 9 pears. [See 22d example. $(12 + 24) \div 4$]

34. The pole is $\frac{4}{5}$ of itself, and as $\frac{3}{5}$ are in the water, the remainder is $\frac{2}{5}$ of the pole; $\frac{1}{2}$ of $\frac{2}{5}$ is $\frac{1}{5}$, hence $\frac{1}{5}$ is in the mud and $\frac{1}{5}$ above the water. If 4 feet is $\frac{1}{5}$ of the pole, $\frac{5}{5}$, or the whole pole, will be 5 times 4 feet, which are 20 feet.

36. If \$ 2 is $\frac{1}{4}$ of what he had left, $\frac{4}{4}$, or all he had left, will be 5 times \$ 2, which are \$ 10; if he spent $\frac{6}{11}$ of his money, he had $\frac{5}{11}$ left, hence $\frac{5}{11}$ of his money is \$ 10. If \$ 10 is $\frac{5}{11}$ of his money, $\frac{1}{11}$ will be $\frac{1}{5}$ of \$ 10, which is \$ 2, and $\frac{11}{11}$, or all his money, will be 11 times \$ 2, which are \$ 22.

37. As 54 pupils are $\frac{9}{10}$ of the second class, $\frac{1}{10}$ will be $\frac{1}{9}$ of 54 pupils, which is 6 pupils, and $\frac{10}{10}$, or the whole class, will be 10 times 6 pupils, which are 60 pupils. As 60 pupils are $\frac{8}{8}$ of twice the third class, $\frac{1}{8}$ will be $\frac{1}{8}$ of 60 pupils, which is 10 pupils, and $\frac{8}{8}$, or twice the third class, will be 5 times 10 pupils, which are 50 pupils; as 50 pupils are 2 times the third class, the third class will be $\frac{1}{2}$ of 50 pupils, which is 25 pupils. Ans. 2d class 60, and 3d class 25 pupils.

Most of the other examples of this Lesson are performed like those of Lesson XXVII., substituting denominate for abstract numbers. The answers of some of the more difficult, not explained above, will be found on the following page.

11. $2.
13. 25 and 5 chickens.
14. 6 sheep.
15. 2 cherries.
16. He received the same.
20. $45.
24. 7 qts. and $\frac{3}{4}$ of a qt.
27. 16 sheep.
30. 30 minutes.
31. $5.
32. 9 answers.
35. 21 quarts.

## LESSON XXIX.—Pages 66–69.

10. First find $\frac{1}{5}$, or divide into 5 equal parts, and then take three of those parts.

15. Reduce 5 to fourths. Ans. 20 times $\frac{1}{4}$.

24. That is, how many whole ones in $\frac{2}{4}$ and $\frac{1}{4}$? Ans. 2 times 1; and 2 times 1 and $\frac{3}{4}$ of 1.

65. The truth of this principle may be shown by dividing a line or an apple into fractional parts. Take the fractions $\frac{1}{3}$ and $\frac{1}{2}$, for instance. Since it takes twice as many sixths as thirds to make a unit, thirds of any unit must be twice as large as sixths of the same unit; hence, if we take one half as many thirds as we take sixths, the value will be the same.

## LESSON XXX.—Pages 69–71.

14. We take 1 from the 3 yards and reduce it to *fourths*. 3 yards $= 2 + \frac{4}{4}$ yards. Taking away $\frac{3}{4}$, we have $2\frac{1}{4}$ yards.

23. $2\frac{7}{12}$.
24. $10\frac{9}{10}$.
25. $12\frac{3}{4}$ bushels.
26. $7\frac{1}{3}$.
27. $24\frac{1}{3}$.
28. 19.
29. $4\frac{3}{5}$.
30. $4\frac{1}{2}$.
31. $1\frac{1}{12}$ years.
32. 50 cts.
33. $7\frac{3}{5}$.
34. $7\frac{1}{2}$.

## LESSON XXXI.—Pages 71–73.

5. 10 times 3 miles are 30 miles, and 10 times $\frac{1}{2}$ are $\frac{10}{2} = 5$ miles, which added to 30 miles make 35 miles.

*6.* $7 \times \$5 = \$35$, and $7 \times \$\frac{1}{2} = \$\frac{7}{2} = \$3\frac{1}{2}$; $\$35 + \$3\frac{1}{2} = \$38\frac{1}{2}$.

| | | |
|---|---|---|
| 14. $\$1\frac{1}{4}$. | 32. 2. | 35. 17 cords. |
| 31. 12; 12. | 33. 4. | 36. 3 loads. |

## LESSON XXXII.—Pages 73–75.

2. In 1 there are $\frac{4}{4}$, and in 9 there are 9 times $\frac{4}{4}$, which are $\frac{36}{4}$; at $\$\frac{3}{4}$ a day, it will take him as many days to earn $\$\frac{36}{4}$, as $\frac{3}{4}$ is contained times in $\frac{36}{4}$, which are 12.

Ans. 12 days.

3. Reduce both to halves. $\frac{77}{2} \div \frac{11}{2} = 7$.

22. $5\frac{1}{7} = \frac{36}{7}$; $2 \times 4\frac{3}{7} = 8\frac{6}{7} = \frac{62}{7}$; $\frac{62}{7} \div \frac{36}{7} = 1\frac{26}{36} = 1\frac{13}{18}$.

23. $5\frac{1}{9} = \frac{46}{9}$; $8 \times 6\frac{2}{9} = 18\frac{6}{9} = \frac{168}{9}$; $\frac{168}{9} \div \frac{46}{9} = 3\frac{30}{46} = 3\frac{15}{23}$.

24. $6 \times 4\frac{4}{7} = 28$; $28 \div 7 = 4$.

25. $10\frac{1}{2} = \frac{21}{2}$; $7 \times 9\frac{1}{2} = 66\frac{1}{2} = \frac{133}{2}$; $\frac{133}{2} \div \frac{21}{2} = 6\frac{7}{21} = 6\frac{1}{3}$.

26. $33\frac{1}{3} = \frac{100}{3}$; $6 \times 16\frac{2}{3} = 100 = \frac{300}{3}$; $\frac{300}{3} \div \frac{100}{3} = 3$.

27. $3 \times 2\frac{1}{5} = 6\frac{3}{5} = \frac{33}{5}$; $10\frac{3}{5} = \frac{53}{5}$; $\frac{53}{5} \div \frac{33}{5} = 1\frac{20}{33}$.

28. $16 \times \frac{1}{8} = \frac{16}{8} = 2 = \frac{12}{6}$; $\frac{12}{6} \div \frac{1}{6} = 12 =$ no. dozen.

29. 1 pound costs $\$\frac{1}{4}$, and 10 pounds cost $\$\frac{10}{4} = \$2\frac{1}{2}$.

30. $7\frac{1}{5} = \frac{36}{5}$; $\$\frac{36}{5} \div \$\frac{4}{5} = 9 =$ no. pupils.

31. $\$6\frac{3}{4} \times 10 = \$67\frac{1}{2} = \$\frac{270}{4}$; $\$2\frac{1}{4} = \$\frac{9}{4}$; $\$\frac{270}{4} \div \$\frac{9}{4} = 30$. Or, the lambs cost $\frac{1}{3}$ as much apiece as the calves, hence there must be three times as many lambs as calves. $3 \times 10 = 30 =$ no. lambs.

32. $\frac{8}{8} = \frac{1}{3}$; $\frac{1}{3} \times 15 = \frac{15}{3} = \frac{120}{16}$; $\frac{120}{16} \div \frac{5}{16} = 24$.

33. 7 in 37, 5 times and 2 over; $2\frac{8}{10} = \frac{28}{10}$, $\frac{1}{7}$ of $\frac{28}{10} = \frac{4}{10} = \frac{2}{5}$; hence 1 yard costs $\$5\frac{2}{5}$. 8 times 5 are 40; 8 times $\frac{2}{5}$ are $\frac{16}{5} = 3\frac{1}{5}$; $40 + 3\frac{1}{5} = 43\frac{1}{5}$.

34. $6\frac{3}{7}$ miles $\div 2 = 3\frac{1}{7}$ miles; $3\frac{1}{7}$ miles $\times 8 = 25\frac{1}{7}$ miles.

35. $1\frac{8}{8} = \frac{11}{8}$; $15\frac{1}{8} = \frac{121}{8}$; $\frac{121}{8} \div \frac{11}{8} = 11 =$ no. men.

36. $16\frac{1}{2} \times 4 = 65\frac{1}{2}$; $65\frac{1}{2} \div 3 = 21\frac{7}{7} =$ no. days.

37. $\frac{3}{16} \times 4 = \frac{3}{4} =$ price per bushel. $2\frac{1}{16} = 2\frac{1}{4} = \frac{9}{4}$; $\frac{9}{4} \times 10 = \frac{90}{4}$; $\frac{90}{4} \div \frac{3}{4} = 30 =$ no. bushels.

38. 6 men can mow $6 \times 1\frac{1}{6} = \frac{42}{6}$ acres in 1 day, and it would take them as many days to mow $14\frac{2}{3} = \frac{72}{6}$ acres, as $\frac{42}{6}$ acres is contained times in $\frac{72}{6}$ acres, which are 2.

Ans. 2 days.

## LESSON XXXIII.—Pages 75–77.

2. Reduce answers to mixed numbers.

3. Reduce fractions to their lowest terms.

9. $\frac{1}{3}$ of 1 is $\frac{1}{3}$, and $\frac{1}{3}$ of 7 is 7 times $\frac{1}{3}$, which are $\frac{7}{3}$; since $\frac{1}{3}$ of 7 is $\frac{7}{3}$, $\frac{2}{3}$ will be 2 times $\frac{7}{3}$, which are $\frac{14}{3}$. Since there are $\frac{3}{3}$ in one whole one, in $\frac{14}{3}$ there will be as many whole ones as $\frac{3}{3}$ is contained times in $\frac{14}{3}$, which are $4\frac{2}{3}$.

16. $\frac{1}{5}$ of 4 oranges, which is $\frac{4}{5}$ of 1 orange.

22. If \$ 33 is $\frac{7}{8}$ of the cost, $\frac{1}{8}$ will be $\frac{1}{7}$ of \$ 33, which is $\frac{33}{7}$ = \$ $4\frac{5}{7}$, and $\frac{8}{8}$, or the whole cost, will be 6 times \$ $4\frac{5}{7}$; $6 \times 4 = 24$, $6 \times \frac{5}{7} = \frac{30}{7} = 4\frac{2}{7}$, \$ 24 + \$ $4\frac{2}{7}$ = \$ $28\frac{2}{7}$.

Where the numerator of an improper fraction is inconveniently large, it is better to reduce to a mixed number before multiplying by another number. If the numerator is so small that it can readily be multiplied mentally, as in the 9th example, it is better to use the improper fraction without reducing.

26. If 3 eggs cost 7 cents, 1 egg will cost $\frac{1}{3}$ of 7 cents, which is $\frac{7}{3}$ of a cent, and 12 eggs will cost $12 \times \frac{7}{3} = \frac{84}{3} =$ 28 cents; 2 dozen will cost $2 \times 28 = 56$ cents. Or,

2 dozen = 24, which is 8 times 3; hence 2 dozen eggs will cost $8 \times 7$ cents = 56 cents.

| | | |
|---|---|---|
| 23. \$ $126\frac{1}{2}$. | 32. \$ 110. | 38. 10. |
| 24. \$ 130. | 33. 60 books. | 39. $24\frac{1}{2}$. |
| 25. $49\frac{1}{5}$ years. | 34. 14. | 40. $27\frac{3}{7}$. |
| 27. \$ $15\frac{3}{4}$. | 35. 8. | 41. 27. |
| 28. $61\frac{3}{5}$ cents. | 36. $52\frac{1}{2}$. | 42. \$ $56\frac{8}{9}$. |
| 29. \$ $72\frac{1}{3}$. | 37. 54. | 45. $3\frac{1}{3}$. |

43. 80 cents a peck will be 10 cents a quart, and 3 quarts *will be worth 30* cents. $\frac{1}{4}$ of 30 cents is $7\frac{1}{2}$ cents.

## LESSON XXXIV.—Pages 77–80.

19. The common denominator must be such that it will contain each of the given denominators.

29. That is, how many units, or whole ones. Ans. $2\frac{1}{56}$.

40. $11\frac{1}{3}$. [See 14th example, Lesson XXX.]

42. Both numbers may be reduced to improper fractions before subtracting; but it is easier to subtract the whole numbers and fractions separately. Ans. $9\frac{17}{40}$.

24. $1\frac{7}{20}$ bushels.
25. $1\frac{18}{35}$.
26. $1\frac{7}{12}$.
27. $2\frac{7}{9}$.
28. $1\frac{40}{88}$.
30. $1\frac{20}{33}$.
31. $4\frac{5}{14}$.
32. $21\frac{3}{8}$.
33. $2\frac{7}{15}$.
34. $1\frac{17}{20}$.
35. $4\frac{4}{9}$.
36. $12\frac{3}{8}$.
37. $\frac{1}{3}$.
38. $\frac{5}{12}$.
39. $\frac{19}{66}$.
41. $7\frac{2}{3}$.
43. $6\frac{5}{6}$.
44. $1\frac{1}{4}$.
45. $14\frac{1}{3}$.
46. $\frac{5}{12}$.
47. $\frac{3}{8}$.

48. $\frac{1}{5}+\frac{2}{3}+\frac{3}{4}=\frac{37}{60}=1\frac{37}{60}$; $2-1\frac{37}{60}=\frac{23}{60}$.

51. Explained like the 65th, Lesson XXIX.

52. $\frac{2}{3}$ of $31=20\frac{2}{3}$; $25-20\frac{2}{3}=4\frac{1}{3}$.

53. $\frac{2}{4}$ of $26=19\frac{1}{2}$; $19\frac{1}{2}-16\frac{1}{3}=3\frac{1}{6}$.

55. His age was $\frac{6}{6}$ of itself. $\frac{6}{6}+\frac{1}{3}+\frac{1}{6}=\frac{8}{6}=\frac{4}{3}$; if $\frac{4}{3}$ of his age was 40 years, $\frac{1}{3}$ was $\frac{1}{4}$ of 40 years $=13\frac{1}{3}$ years, and $\frac{3}{3}$ was $2\times13\frac{1}{3}=26\frac{2}{3}$ years.

56. $\frac{1}{4}+\frac{2}{5}=\frac{13}{20}$; $\frac{20}{20}-\frac{13}{20}=\frac{7}{20}=$ remainder $=2\times\$9=\$18$; if $\frac{7}{20}$ of his money $=\$18$, $\frac{1}{20}=\$2\frac{4}{7}$, and $\frac{20}{20}=\$51\frac{3}{7}$.

## LESSON XXXV.—Pages 80–82.

1. $\frac{1}{5}=\frac{2}{10}$; $\frac{1}{2}$ of $\frac{2}{10}=\frac{1}{10}$.

2. $\frac{1}{4}=\frac{3}{12}$; $\frac{1}{3}$ of $\frac{3}{12}=\frac{1}{12}$. This principle can also be readily shown by dividing a line into 4 equal parts, and then dividing each one of those parts into 3 other equal parts. It will take 12 of the smaller parts to make the whole line.

10. $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$, and $\frac{1}{3}$ of $\frac{2}{5}=2$ times $\frac{1}{15}=\frac{2}{15}$.

12. $\frac{1}{4}$ of $\frac{1}{3}=\frac{1}{12}$, $\frac{1}{4}$ of $\frac{2}{3}=2\times\frac{1}{12}=\frac{2}{12}=\frac{1}{6}$, $\frac{3}{4}$ of $\frac{3}{4}=8\times\frac{1}{8}=\frac{8}{8}=\frac{1}{2}$.

19. $\frac{1}{2}$ of 33 = 8 and 1 remaining; $1\frac{1}{2} = \frac{1}{2}$, $\frac{1}{4}$ of $\frac{1}{2} = \frac{1}{8}$; hence $\frac{1}{4}$ of $33\frac{1}{2} = 8\frac{1}{8}$.

23. $\frac{1}{7}$ of 11 = 1 and 4 rem., $4\frac{1}{2} = \frac{9}{2}$, $\frac{1}{7}$ of $\frac{9}{2} = \frac{9}{14}$, hence $\frac{1}{7}$ of $11\frac{1}{2} = 1\frac{9}{14}$; $\frac{1}{21}$ of $11\frac{1}{2} = 5 \times 1\frac{9}{14}$, $5 \times 1 = 5$, $5 \times \frac{9}{14} = \frac{45}{14} = 3\frac{3}{14}$, $5 + 3\frac{3}{14} = 8\frac{3}{14}$.

20. $2\frac{1}{8}$.
21. $2\frac{1}{12}$.
22. $1\frac{1}{2}$; $2\frac{1}{4}$; $\frac{7}{8}$; $8\frac{3}{8}$.
23. $8\frac{3}{14}$; $\frac{9}{10}$; $1\frac{27}{40}$; $2\frac{13}{27}$.
24. $\$ 1\frac{7}{10}$.
25. $7\frac{1}{32}$ cents.
26. $8\frac{1}{3}$ hours.
27. $\$ \frac{25}{48}$.
28. $\frac{4}{5}$ bushel.

29. $\frac{1}{8}$ of $\$ \frac{7}{8} = \$ \frac{7}{64}$ = cost of 1 quart; $3 \times \$ \frac{7}{64} = \$ \frac{21}{64}$ = cost of 3 quarts.

30. $\frac{1}{8}$ of $\$ \frac{1}{8} = \$ \frac{1}{64}$ = cost of 1 pound; $5 \times \$ \frac{1}{64} = \$ \frac{5}{64}$ = cost of 5 pounds.

31. $\frac{1}{8}$ of $\$ 31\frac{1}{2} = \$ 10\frac{1}{2}$ = cost of 1 ton; $\frac{2}{3}$ of $\$ 10\frac{1}{2} = \$ 7\frac{1}{3}$ = cost of $\frac{2}{3}$ of a ton.

32. $\frac{1}{8}$ of $\$ 8\frac{1}{7} = \$ 1\frac{1}{7} = 1$ day's wages; $\frac{1}{4}$ of $\frac{1}{7} = \frac{1}{7}$; $\frac{1}{7}$ of $\$ 1\frac{1}{7} = \$ \frac{10}{49} = \frac{1}{7}$ of a day's wages.

33. $\frac{2}{3}$ of $\$ 6\frac{1}{2} = \$ 2\frac{1}{3}$ = cost of 1 yard; $\frac{3}{11}$ of $\$ 2\frac{1}{3} = \$ \frac{7}{11}$ = cost of $\frac{3}{11}$ of a yard.

34. $\frac{1}{7}$ of $5\frac{1}{2} = \frac{11}{7}$; $\frac{11}{7}$ of $\frac{8}{3}$ dimes $= \frac{88}{21} = 4\frac{4}{21}$ dimes.

35. 20 — 5 = 15 years $= \frac{3}{4}$ of $\frac{4}{5} = \frac{12}{20}$ of Walter's age; $\frac{1}{20} = \frac{1}{15}$ of 15 years = 1 year, and $\frac{28}{20} = 28 \times 1$ year $= 28$ years = Walter's age.

## LESSON XXXVI.—Pages 82-84.

4. $9 = \frac{63}{7}$; $\frac{4}{7}$ is contained in $\frac{63}{7}$, $15\frac{3}{4}$ times.

It will be seen that, when the denominators are the same, they have no effect upon the result. 2 elevenths is contained in 10 elevenths 5 times, in the same way that 2 apples is contained in 10 apples 5 times.

7. $\frac{35}{3} \div \frac{7}{3} = 5\frac{1}{7}$; $\frac{21}{3} \div \frac{8}{3} = 3\frac{1}{8} = 3\frac{1}{2}$.

13. $5\frac{5}{16}$.
14. $5\frac{1}{3}$ weeks.
15. $1\frac{1}{6}$.
16. 2 pounds.
17. $2\frac{11}{12}$.
18. $13\frac{5}{12}$.
19. $2\frac{23}{36}$.
20. $19\frac{1}{9}$.
21. $2\frac{41}{200}$.
22. $2\frac{1}{2}$.
23. $1\frac{147}{256}$.
24. 4 lambs.
25. $7\frac{1}{2}$ yards.
26. $\$ \frac{7}{16}$.
27. 14 bushels.

28. $5 \times \frac{2}{3} = \frac{10}{3}$ miles in 1 hour. It would take him as many hours as $\frac{10}{3}$ miles is contained times in $\frac{25}{3}$ miles, which are $2\frac{1}{2}$. Ans. $2\frac{1}{2}$ hours.

29. $\frac{3}{4}$ of $\$\frac{7}{2} = \$\frac{21}{8}$; $\frac{21}{8} \div \frac{1}{12} = \frac{252}{24} \div \frac{2}{24} = 31\frac{1}{2} =$ no. lbs. of sugar.

30. If $\frac{3}{5}$ of a yard cost $\$\frac{3}{20}$, $\frac{1}{5}$ will cost $\frac{1}{3}$ of $\$\frac{3}{20}$, which is $\$\frac{1}{20}$, and a yard will cost 5 times $\$\frac{1}{20} = \$\frac{5}{20}$; $1\frac{1}{4} = \frac{5}{4}$; $\frac{5}{4}$ of a yard will cost $\frac{5}{4}$ of $\$\frac{5}{20} = \$\frac{25}{80}$.

31. $\frac{3}{5}$ of $\$\frac{15}{8} = \$\frac{9}{8} =$ money spent for tea; $\frac{9}{8} \div \frac{3}{4} = \frac{18}{4} \div \frac{3}{4} = 6 =$ no. lbs. of tea. $\$\frac{15}{8} - \$\frac{9}{8} = \$\frac{6}{8} =$ money spent for coffee; $\frac{6}{8} \div \frac{1}{6} = \frac{18}{6} \div \frac{1}{6} = 18 =$ no. lbs. of coffee.

32. If $\frac{3}{4}$ of a barrel cost $\$\,3\frac{3}{4}$, $\frac{1}{7}$ will cost $\frac{1}{3}$ of $\$\,3\frac{3}{4}$, which is $\$\frac{15}{8}$, and $\frac{7}{7}$ will cost $7 \times \$\frac{15}{8} = \$\frac{105}{8} =$ cost of 1 barrel; $\frac{3}{3}$ of a barrel will cost $\frac{3}{3}$ of $\$\frac{105}{8} = \$\frac{315}{24} = \$\,8\frac{3}{24}$.

33. $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$; $\frac{1}{7}$ of $\frac{21}{4} = 3$; $3 \div \frac{1}{4} = \frac{12}{4} \div \frac{1}{4} = 12$.

34. $5 \times \$\frac{7}{8} = \$\frac{35}{8} =$ value of 5 bushels of corn; $\frac{25}{8} \div \frac{5}{16} = \frac{70}{16} \div \frac{5}{16} = 14 =$ no. bushels of oats.

## LESSON XXXVII.—Pages 84–86.

16. 1 peck = 8 quarts, and 5 quarts is $\frac{5}{8}$ of 8 quarts; therefore 5 quarts is $\frac{5}{8}$ of 1 peck.

18. Reduce both to half-pounds. $2\frac{9}{2}$ pounds is $\frac{29}{50}$ of $\frac{50}{2}$ pounds; therefore $14\frac{1}{2}$ pounds is $\frac{29}{50}$ of 1 quarter.

21. Reduce both to quarts. Ans. $\frac{80}{160} = \frac{8}{16}$ of 5 bushels.

28. 5 E. E. 3 qr. = 28 qr. = 7 yds. Ans. $\frac{7}{8}$ of 8 yards

9. $\frac{1}{15}$; $\frac{4}{35}$.
10. $\frac{1}{36}$; $\frac{1}{12}$.
11. $\frac{24}{7}$; $\frac{30}{5}$.
12. $\frac{7}{18}$; $\frac{9}{35}$.
13. $\frac{5}{27}$; $\frac{11}{33}$.
14. $\frac{27}{33}$; $\frac{14}{15}$.
15. $\frac{12}{5}$; $\frac{9}{11}$.
17. $\frac{9}{20}$.
19. $\frac{12}{21}$.
20. $\frac{5}{11}$.
22. $\frac{12}{16}$.
23. $\frac{40}{56}$.

24. $\frac{1}{4}$.
25. $\frac{1}{8}$.
26. $\frac{9}{16}$.
27. $\frac{21}{64}$.
30. $\frac{3}{4}$ inch.
31. $\frac{5}{8}$ pwt.
32. $\frac{4}{5}$ rod.
33. $\frac{14}{15}$ day.
34. $\frac{24}{64}$ gill.
36. $\frac{3}{21}$ week.
37. $\frac{1}{1120}$ yard.

38. $\frac{1}{8000}$ ton.
39. $\frac{1}{96}$ bushel.
40. $\frac{1}{480}$ pound.
41. 21 gallons.
43. 4 oz. 10 pwt.
44. 2 ft. 6 in.
45. 47 gal. 1 qt.
46. 2 pk. $2\frac{1}{4}$ qt.
47. 3 cwt. 3 qr.
48. 1d. 21h. 49m. $5\frac{5}{11}$s.

## LESSON XXXVIII. — Pages 86 - 89.

3. $\frac{2}{5}$ of 60 = 24 = no. rods John runs in 1 minute. $15\frac{1}{5} \div 24 = \frac{76}{5} \div \frac{120}{5} = \frac{76}{120} = \frac{38}{60}$ = part of a minute it will take John. Reducing to seconds, we have 38 seconds. Or, James runs 1 rod a second, and John $\frac{2}{5}$ of a rod a second. $\frac{76}{5} \div \frac{2}{5} = 38$ = no. seconds it will take John.

10. A's part of the proceeds is $2 \times \$24 = \$48 = \frac{4}{9}$ of the whole; hence $\frac{9}{9} = \$108$ = what the wood sold for. $\$108 - \$48 = \$60$ = B's part.

11. $20 + 4 = 24 = \frac{3}{5}$ of no. scholars; hence $\frac{1}{5} = 40$ = no. scholars. $\frac{2}{5}$ of $40 = 16$; $16 + 4 = 20$ = no. boys.

14. $\$60 = \frac{5}{6}$ of the cost; $\frac{1}{6} = \$10$ = gain.

15. $\frac{4}{7}$ of it sold for $\$28$; $\frac{1}{7}$ would sell for $\$98$; $\$98 - \$64 = \$34$ = gain.

16. $4 \times 3$ ft. = 12 ft. = part in the ground = $\frac{1}{5}$ of the pole; $\frac{4}{5} = 60$ ft. = length of the pole.

17. $\frac{3}{4}$ of $\frac{5}{6} = \frac{5}{12}$; $\frac{7}{12}$ = remainder; $\frac{5}{12}$ of $\frac{7}{4} = \frac{5}{21}$ = Smith's part of the whole = $\$15$; $\frac{21}{21} = \$63$ = whole amount. $\frac{3}{7}$ of $\$63 = \$27$ = Brown's share. $\$63 - \$15 - \$27 = \$21$ = Robinson's share.

19. We may solve this in the two usual ways (1st, by finding how much $\$1$ will purchase, and, 2d, by finding the cost of 1 barrel); but there is a simpler process. As $\$4\frac{1}{2}$ is 2 times $\$2\frac{1}{4}$, 2 times $\frac{3}{7} = \frac{6}{7}$ of a barrel can be purchased for $\$4\frac{1}{2}$.

23. $\frac{5}{8}$ of $\frac{8}{8} = \frac{5}{16}$ = part contributed by John, and $\frac{16}{16} - \frac{5}{16} = \frac{11}{16}$ = part contributed by Richard. If $\frac{5}{16}$ of the gain = $\$60$, $\frac{1}{16} = \$12$, and $\frac{11}{16} = \$132$ = Richard's share of the gain.

24. $\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$; $\frac{12}{12} - \frac{11}{12} = \frac{1}{12} = 4$ cents; $\frac{12}{12} = 48$ cents = whole money. $\frac{1}{4}$ of 48 cents = 12 cents; 12 cents $\div$ 2 cents = 6 = no. pencils.

| | | |
|---|---|---|
| 1. $\$6$. | 5. 66. | 8. $4\frac{2}{3}$. |
| 2. $\$40$. | 6. $\$10\frac{1}{2}$. | 9. $\$3$. |
| *4. 36.* | 7. 4. | 12. 63. |

13. 40.
18. 54.
20. $5\frac{1}{3}$ dozen.
21. 6.
22. 5.
25. 4.
26. 6.
27. 35. [$\frac{3}{5} = 21$]
28. $2\frac{1}{2}$.
29. 4.
30. \$ $8\frac{3}{5}$. [$\frac{1}{7} = 1\frac{1}{5}$]
31. \$ $4\frac{1}{5}$.

32. At \$ $\frac{1}{12}$ a quart, 1 bushel costs \$ $\frac{8}{8}$; \$ $\frac{7}{8}$ is $\frac{7}{8}$ of \$ $\frac{8}{8}$; hence, for \$ $2\frac{1}{2}$, $\frac{7}{8}$ of a bushel can be bought.

33. $6 \times 10$ hours $= 60$ hours, in 1 week; $60 \times \$\frac{1}{5} = \$12 = 1$ week's pay; \$ 7 is $\frac{7}{12}$ of \$ 12; hence it would take $\frac{7}{12}$ of a week to earn \$ 7.

34. $\frac{5}{4}$ of a peck $= \frac{5}{16}$ of a bushel; $\frac{3}{16} + \frac{5}{16} = \frac{1}{2}$ bushel.

35. $\frac{1}{4}$ of a quarter $= \frac{1}{16}$ of a cwt.; $\frac{5}{8} - \frac{1}{16} = \frac{9}{16}$ of a cwt.

36. \$ 1000 — \$ 100 = \$ 900 = $\frac{3}{4}$ of his capital; $\frac{1}{4} =$ \$ 300 = amount lost.

37. If $\frac{8}{7} = 60$, $\frac{7}{7} = 70$; $\frac{1}{8}$ of $30 = \frac{30}{8}$; $70 \div \frac{30}{8} = 18\frac{8}{8}$.

38. If $\frac{7}{8} = 84$, $\frac{8}{8} = 96$; $\frac{1}{2}$ of $22 = 11$; $96 \div 11 = 8\frac{8}{11}$.

39. $\frac{3}{4}$ of \$ 48 = \$ 36; if $\frac{3}{5}$ = \$ 36, $\frac{5}{5}$ = \$ 162 = the cost of the watch.

## LESSON XXXIX.—Pages 89–91.

1. If $\frac{3}{7} = 7$, $\frac{5}{5} = \frac{35}{5}$; $\frac{1}{7}$ of $\frac{35}{5} = \frac{5}{5} = 1\frac{3}{5}$.
2. If $\frac{4}{3} = 10$, $\frac{3}{3} = 22\frac{1}{2}$; $\frac{2}{3}$ of $22\frac{1}{2} = 9$.
3. If $\frac{5}{6} = 12$, $\frac{6}{6} = \frac{72}{6}$; $\frac{5}{6}$ of $\frac{72}{5} = \frac{27}{5} = 5\frac{2}{5}$.
4. $2 \times \frac{3}{5} = \frac{4}{5}$; $\frac{4}{5}$ of $37 = 16\frac{4}{5}$; $18 - 16\frac{4}{5} = 1\frac{1}{5}$.
5. $5 \times \frac{3}{7} = \frac{15}{7}$; $\frac{15}{7}$ of $30 = 64\frac{2}{7}$; $65 - 64\frac{2}{7} = \frac{5}{7}$.
6. $4 \times \frac{5}{6} = \frac{20}{6} = \frac{10}{3}$; $\frac{10}{3}$ of $19 = 63\frac{1}{3}$; $63\frac{1}{3} - 50\frac{1}{3} = 13$.
7. $3 \times \frac{5}{5} = 5$; 5 times $22 = 110$; $110 - 100 = 10$.
8. Itself is $\frac{5}{5}$; $\frac{5}{5} + \frac{3}{5} = \frac{8}{5}$; if $\frac{3}{5} = 48$, $\frac{5}{5} = 30$.
9. $\frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \frac{4}{4}$; if $\frac{4}{4} = 100$, $\frac{1}{4} = 57\frac{1}{7}$.
10. $\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$; $\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$; if $\frac{3}{2} = 99$ y., $\frac{2}{2} = 66$ y.
11. $\frac{1}{3} + \frac{1}{8} + \frac{1}{3} = \frac{18}{15}$; $\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$; if $\frac{2}{15} = 16$ cents, $\frac{15}{15} = 120$ cents.

12. If he spent $\frac{3}{4}$ of the remainder, he had $\frac{1}{4}$ of it left, which must be \$ 100; if $\frac{1}{4} =$ \$ 100, $\frac{4}{4} =$ \$ 400 = the remainder at the end of 6 months. If he spent $\frac{1}{4}$ of the whole in 6 months, there must have been $\frac{3}{4}$ of the whole remaining; if $\frac{3}{4} =$ \$ 400, $\frac{4}{4} =$ \$ $533\frac{1}{3}$ = whole legacy. Or,

$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$; $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$; $\frac{1}{4}$ of $\frac{3}{4} = \frac{3}{16}$ of the whole. If $\frac{3}{16} = \$100$, $\frac{16}{16} = \$533\frac{1}{3}$.

13. If $\frac{3}{4} = \$24$, $\frac{7}{7} = \$84$ = value of A's horse. $\$84 - \$24 = \$60$ = value of B's horse. (Ownership taken *before* exchanging.)

14. $\frac{1}{3}$ of a day.

15. If 3 horses consume $\frac{3}{4}$ of a bushel, 1 horse will consume $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$ of a bushel, and it will take as many horses to consume $3\frac{1}{2} = \frac{7}{2} = \frac{14}{4}$ bushels, as $\frac{1}{4}$ of a bushel is contained times in $\frac{14}{4}$ bushels, which are 14. Or,

4 horses would consume 1 bushel, and it would require $3\frac{1}{2}$ times 4 horses = 14 horses, to consume $3\frac{1}{2}$ bushels.

16. $\frac{2}{9}$ of 36 = 8; if $\frac{1}{7} = 8$, $\frac{7}{7} = 14$; $\frac{1}{10}$ of 20 = 2; $14 \div 2 = 7$.

17. $\frac{3}{4}$ of 44 = 33; if $\frac{3}{8} = 33$, $\frac{8}{8} = 55$; $\frac{1}{7}$ of 35 = 5; $55 \div 5 = 11$.

18. $\frac{4}{5}$ of 30 = 40; if $\frac{5}{5} = 40$, $\frac{9}{9} = 72$; $\frac{1}{9}$ of 45 = 5; $72 \div 5 = 14\frac{2}{5}$.

19. 9 yards $\frac{1}{2}$ a yard wide = $\frac{1}{2}$ of 9 = $4\frac{1}{2}$ yards 1 yard wide; and they would be equal to as many yards $\frac{3}{4}$ of a yard wide, as $\frac{3}{4}$ is contained times in $4\frac{1}{2}$ yards, which are 6.

20. $2 \times 6$ cents = 12 cents, and $2 \times 12$ cents = 24 *cents*, 12 cents + 24 cents = 36 cents; $\frac{1}{8} + \frac{1}{7} + \frac{11}{56} = \frac{23}{56}$, $\frac{33}{56} - \frac{23}{56} = \frac{10}{56}$; if $\frac{10}{56} = 36$ cents, $\frac{56}{56} = 105$ cents = what he had at first.

21. If $\frac{2}{3}$ of the cost = \$11, $\frac{3}{3} = \$16\frac{1}{2}$ = cost of the cart.

22. If he pays $\frac{4}{5}$ of his debts, $\frac{1}{5}$ is left unpaid; if $\frac{1}{5} = \$28$, $\frac{4}{5} = 4$ times \$28 = \$112 = amount received on both days. The amount received Jan. 2d = $\frac{4}{7}$ of itself, and $\frac{3}{4}$ added make $\frac{7}{4}$; therefore \$112 = $\frac{7}{4}$ of what was received Jan. 2d. If $\$112 = \frac{7}{4}$, $\frac{1}{4} = \frac{1}{7}$ of \$112 = \$16; $\frac{4}{4} = 4$ times \$16 = \$64 = amount received Jan. 2d; and $\frac{3}{4} = 3$ times \$16 = \$48 = amount received Jan. 1st.

23. $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$; $\frac{1}{5}$ of $\frac{2}{5} = \frac{2}{10}$ = part borrowed; $\frac{5}{5} + \frac{2}{10} = \frac{9}{10}$ = part he then had; $\frac{10}{10} - \frac{9}{10} = \frac{1}{10} = 6$ cents; if $\frac{1}{10} = 6$ *cents*, $\frac{10}{10} = 60$ cents = amount at first.

*24.* $\frac{4}{5}$ *of* $42 = 30$; if $\frac{3}{3} = 30$, $\frac{5}{5} = 45$; $\frac{1}{5}$ of $13\frac{1}{2} = 9$; $45 \div 9 = 5$.

25. $\frac{1}{4}$ of 16 = 12; if $\frac{1}{4}$ = 12, $\frac{1}{7}$ = 14; $\frac{1}{5}$ of 15 = 6; 14 ÷ 6 = $2\frac{1}{3}$.

26. $\frac{1}{6}$ of 20 = 16; if $\frac{2}{3}$ = 16, $\frac{2}{9}$ = 18; $\frac{1}{3}$ of 56 = $6\frac{2}{3}$; 18 ÷ $6\frac{2}{3}$ = $2\frac{2}{13}$.

27. $\frac{2}{3}$ of 60 = 40; if $\frac{2}{5}$ = 40, $\frac{1}{5}$ = 100; $\frac{4}{5}$ of 40 = 25; 100 ÷ 25 = 4.

28. If 6 persons will consume the flour in 6 months, it will take 6 times 6 persons = 36 persons, to consume it in 1 month; if 36 persons will consume it in 1 month, it will take $\frac{2}{9}$ of 36 persons = 8 persons, to consume it in $4\frac{1}{2}$ months.

29. Richard's money = $\frac{5}{7}$ of \$ 42 = \$ 30. $\frac{6}{5}$ of \$ 30 = \$ 36; if $\frac{3}{4}$ of Henry's = \$ 36, $\frac{1}{4}$ = \$ 48 = Henry's money.

30. $\frac{1}{6} + \frac{2}{3} + \frac{1}{12} = \frac{11}{12}$; $\frac{12}{12} - \frac{11}{12} = \frac{1}{12}$; if $\frac{1}{12}$ = 14 trees, $\frac{12}{12}$ = 168 trees = whole number. $\frac{1}{6}$ of 168 trees = 28 peach-trees, $\frac{2}{3}$ of 168 trees = 112 apple-trees, and $\frac{1}{12}$ of 168 trees = 14 plum-trees.

31. $\frac{1}{3} + \frac{1}{12} + \frac{1}{6} = \frac{7}{12}$; $\frac{12}{12} - \frac{7}{12} = \frac{5}{12}$ = what he had left before losing \$ 18; $\frac{5}{12} + \frac{1}{8}$ = \$ 18 + \$ 8, that is, $\frac{13}{24}$ = \$ 26; if $\frac{1}{24}$ = \$ 26, $\frac{24}{24}$ = \$ 48 = what he had at first.

32. 100 geese − $2\frac{1}{2}$ geese = $97\frac{1}{2}$ geese; $\frac{3}{4} + \frac{1}{2} = \frac{5}{4}$; if $\frac{5}{4}$ of the number = $97\frac{1}{2}$ geese, $\frac{1}{4}$ = $32\frac{1}{2}$, and $\frac{2}{4}$ = 65 geese = the number he had.

33. $3\frac{1}{2}$ times 6 inches = 21 inches = length of the body; 6 inches + $10\frac{1}{2}$ inches = $16\frac{1}{2}$ inches = length of the tail; 6 + 21 + $16\frac{1}{2}$ = $43\frac{1}{2}$ inches = length of the fish.

34. $\frac{1}{3}$ of \$ 24 = \$ 8 = value of the first horse; 6 × \$ 24 = \$ 144 = value of the second horse; \$ 8 + \$ 24 = \$ 32 = value of first horse with saddle; \$ 144 + \$ 24 = \$ 168 = value of second horse with saddle.

## LESSON XL.—Pages 92, 93.

6. 1 per cent., or $\frac{1}{100}$, of \$ 100 is \$ 1, and 2 per cent. is 2 times \$ 1 = \$ 2. 2 per cent. of \$ 10 is 2 dimes, or 20 cents. 2 per cent. of \$ 1 is 2 cents.

7. 6 dimes, or 60 cents; \$ 1; 96 cents; \$ 1.28.

8. 1 per cent. of \$ 320 is 320 cents, or 32 dimes, and 3 per

cent. is 8 times 32 dimes, which are 96 dimes, or $ 9.60. 3 per cent. of $ 460 is $ 13.80.* 3 per cent. of $ 280 is $ 8.40

9. 1 per cent. of 320 bushels is $\frac{320}{100} = 3\frac{20}{100} = 3\frac{1}{5}$ bushels and 5 per cent. is 5 times $3\frac{1}{5}$ bushels = 16 bushels. Or, $\frac{5}{100} = \frac{1}{20}$, and $\frac{1}{20}$ of 320 bushels = 16 bushels.

10. 1 per cent. = $2\frac{1}{4}$ tons, and 4 per cent. = 9 tons. Or, $\frac{1}{25}$ of 225 tons = 9 tons.

11. 6 lbs.; 9 lbs.; $21\frac{18}{25}$ lbs.
12. $24\frac{9}{25}$ yards.
13. $ $56\frac{1}{4}$; $ 112.
14. $ 150; $ 375.
15. $28\frac{1}{4}$ barrels.
16. 35 per cent.
17. Nothing.
18. 67 per cent.
19. 77 per cent.

20. 36 per cent. = $\frac{9}{25}$; $\frac{1}{25}$ of $ 70 = $ $2\frac{4}{5}$, and $\frac{9}{25} = 9 \times$ $ $2\frac{4}{5} =$ $ $25\frac{1}{5} =$ $ 25.20.

21. 28 per cent. = $\frac{7}{25}$; $\frac{7}{25}$ of $ 50 = $ 14.

22. 48 per cent. = $\frac{12}{25}$; $\frac{1}{25}$ of $ 80 = $ $3\frac{1}{4}$, and $\frac{12}{25} = 12 \times$ $ $3\frac{1}{4} =$ $ $38\frac{3}{4} =$ $ 38.40.

23. 45 per cent. = $\frac{9}{20}$; $\frac{9}{20}$ of $ 60 = $ 27.

24. 1 per cent. of $ 100 = $ 1, and $\frac{1}{2}$ per cent. = $ $\frac{1}{2}$ = 5 dimes, or 50 cents. 1 per cent. of $ 10 = 10 cents, and $\frac{1}{2}$ per cent. = $\frac{1}{2}$ of 10 cents = 5 cents. 1 per cent. of $ 1 = 1 cent, and $\frac{1}{2}$ per cent. = $\frac{1}{2}$ cent = 5 mills.

25. $ 1.50; 15 cents; $\frac{3}{4}$ of a cent, or $7\frac{1}{2}$ mills.
26. 24 pounds.
27. $67\frac{1}{2}$ yards.
28. $ 12.50.
29. 50 cents.
30. $ 3.
31. 54; 55.

## LESSON XLI.—Pages 94, 95.

2. $\frac{1}{8}$.
3. $\frac{1}{3}$.
4. $\frac{1}{4}$.
5. $\frac{6}{25}$.
6. $\frac{11}{25}$.
8. 16 per cent.
9. $12\frac{1}{2}$ per cent.
11. 20 per cent.
12. 60 per cent.
13. 50 per cent.
14. $62\frac{1}{2}$ per cent.
15. $62\frac{1}{2}$; 24; 15.
16. 20 per cent = $\frac{1}{5}$; $\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$ = remainder; $\frac{3}{5}$ of $\frac{4}{5}$

* If the number of cents or dimes is found to be inconveniently *large, dollars and* fractions of a dollar may be used. 1 per cent. of $460 = $4\frac{60}{100} =$ $4\frac{3}{5}$, and 3 per cent. $= 3 \times$ $4\frac{3}{5} =$ $13\frac{4}{5} =$ $13.80

36. At the end of the 1st year he has 120 per cent.; $\frac{120}{100} = \frac{6}{5}$; at the end of the 2d year he has $\frac{6}{5}$ of 120 per cent. = 144 per cent.; at the end of the 3d year he has $\frac{6}{5}$ of 144 per cent. = $172\frac{4}{5}$ per cent.; hence he has $72\frac{4}{5}$ per cent. more than when he began.

## LESSON XLIII.—Pages 99–101.

5. Since the interest of $1 for 1 year, at 6 per cent., is 6 cents, for 2 years it will be 2 times 6 cents, which are 12 cents; since the interest of $1 for 2 years is 12 cents, the interest of $4 will be 4 times 12 cents, which are 48 cents.

The interest of $6 will be 6 times 12 cents, which are 72 cents. The interest of $8 = 8 × 12 cents = 96 cents.

12. Since the interest of $100 for 1 year, at 7 per cent., is $7, for 5 years it will be 5 times $7, which are $35.

For 6 years it is $42; for 7 years it is $49.

13. 150 = $1\frac{1}{2}$ hundred. Since the interest of $100 for 1 year, at 7 per cent., is $7, the interest of $1\frac{1}{2}$ hundred dollars for 1 year will be $1\frac{1}{2}$ times $7, which are $10\frac{1}{2}$, and for 2 years will be 2 times $10\frac{1}{2}$, which are $21.

The interest for 3 years is 3 × $10\frac{1}{2}$ = $31\frac{1}{2}$.

14. Since the interest of $1 for 1 year, at 7 per cent., is 7 cents, for $106 it will be 106 times 7 cents, which are 742 cents, and for 3 years it will be 3 times 742 cents, which are $22.26.

If the interest at 7 per cent. is $22.26, at 1 per cent. it will be $\frac{1}{7}$ of $22.26, which is $3.18, and at 8 per cent. it will be 8 times $3.18, which are $25.44.

6. $1.80; $2.16; $3.60.
7. $3; $12.
8. $25.20; $32.40.
9. 6 cts.; 7 cts.; 5 cts.
10. 60 cts.; 70 cts.; 50 cts.
11. $6; $7; $5.
15. $128; $153.60.
16. $145.25; $189.
18. 5 cts.; 5 mills; 3 cts.
19. 60 cts.; 9 cts.; 90 cts.
20. $1; $1.50; $1.85; $2.10; $2.13.
21. $2; $2.50; $3; $4; $5.
22. $3.50; $4.
23. $26; $52; $78; $13; $91.
24. $15.75.

25. $59.25. | 26. $72.83$\frac{1}{2}$.

27. 625 = 6$\frac{1}{4}$ hundred. The interest of $100 for 1 year, at 7 per cent., is $7, and for 5 years it will be 5 times $7, which are $35; 3 mo. = $\frac{1}{4}$ of a year, hence the interest of $100 for 3 mo., at 7 per cent., will be $\frac{1}{4}$ of $7, which is $1$\frac{3}{4}$; $35 + $1$\frac{3}{4}$ = $36$\frac{3}{4}$; if the interest of $100 for 5 years and 3 months is $36$\frac{3}{4}$, the interest of 6$\frac{1}{4}$ hundred dollars will be 6$\frac{1}{4}$ times $36$\frac{3}{4}$, which are $229$\frac{11}{16}$.

## LESSON XLIV.—Pages 101–103.

3. Since the interest of $1 for 1 year, at 6 per cent., is 6 cents, for 1 month it will be $\frac{1}{12}$ of 6 cents, which is $\frac{1}{2}$ a cent, or 5 mills; 5 days = $\frac{1}{6}$ of a month, hence the interest of $1 for 5 days, at 6 per cent., is $\frac{1}{6}$ of 5 mills, which is $\frac{5}{6}$ of a mill; since the interest of $1 is $\frac{5}{6}$ of a mill, the interest of $20 will be 20 times $\frac{5}{6}$ of a mill, which are 16$\frac{2}{3}$ mills = 1 cent 6$\frac{2}{3}$ mills.

For 12 days it is 4 cents; for 15 days it is 5 cents.

The most difficult examples in Lessons XLIV. and XLV. can be omitted by those teachers who do not wish to train their pupils to use large numbers in mental arithmetic.

2. 5 mills; 1 cent; 2 cts.; 3$\frac{1}{3}$ mills; 6$\frac{2}{3}$ mills.
4. 20 cts. 4$\frac{1}{6}$ mills; 23$\frac{1}{3}$ cts.; 43$\frac{3}{4}$ cts.
5. $9.60; 80 cts.; $10.40; 13$\frac{1}{3}$ cts.; $10.53$\frac{1}{3}$.
6. $49.55$\frac{5}{6}$.
7. $2.62$\frac{1}{2}$.
8. $36.70$\frac{5}{6}$.
9. $24.20.
10. $8.86$\frac{2}{3}$.
11. $9; $12.60.
12. $2.97$\frac{1}{2}$; $3.40.
13. $48.
14. $45.86$\frac{2}{3}$.
15. $12.72.
16. $31.36.
17. $26.88.
18. $46.02$\frac{1}{2}$.
19. $49.
20. $7.98; $5.32; $6.65; $9.31; $10.64.
21. $13.77; $9.18; $11.47$\frac{1}{2}$; $16.06$\frac{1}{2}$; $18.36.
22. $32.24; $26.86$\frac{2}{3}$; $42.98$\frac{2}{3}$; $37.61$\frac{1}{3}$.
23. $20.33$\frac{1}{3}$; $30.50.
24. $24. [(18 ÷ 6) × 8]
25. $13$\frac{1}{2}$; $3$\frac{5}{8}$; $2$\frac{1}{4}$; $1$\frac{1}{8}$.
26. $2.40; $1.20; 80 cts.; 40 cts.

## LESSON XLV.—Pages 103, 104.

Sometimes the solution can be simplified by reducing the months and days to fractions of a year, as in the following example.

13. 2 months and 20 days = 80 days = $\frac{80}{360} = \frac{2}{9}$ of a year; hence we have $2\frac{2}{9}$ years. The interest of \$ 100 for 1 year, at $3\frac{1}{2}$ per cent., is \$ $3\frac{1}{2}$, and for $2\frac{2}{9}$ years it will be $2\frac{2}{9}$ times \$ $3\frac{1}{2} =$ \$ $\frac{140}{18} =$ \$ $7\frac{7}{9}$; if the interest of \$ 100 for $2\frac{2}{9}$ years is \$ $7\frac{7}{9}$, the interest of \$ 400 for the same time will be 4 times \$ $7\frac{7}{9}$, which are \$ $31\frac{1}{9}$; \$ 400 + \$ $31\frac{1}{9} =$ \$ $431\frac{1}{9} =$ amount.

| | | |
|---|---|---|
| 2. \$ $69\frac{2}{5}$. | 10. \$ $192\frac{7}{9}$. | 19. \$ 246. |
| 3. \$ 92. | 11. \$ $217\frac{1}{3}$. | 20. \$ $181.62\frac{2}{3}$. |
| 4. \$ 124. | 12. \$ $217\frac{1}{2}$. | 21. \$ $224.57\frac{19}{48}$. |
| 5. \$ $29\frac{2}{5}$. | 14. \$ $192.53\frac{1}{3}$. | 22. \$ 154.91. |
| 6. \$ $142\frac{1}{2}$. | 15. \$ $230\frac{7}{15}$. | 23. \$ 232. |
| 7. \$ $23\frac{1}{3}$. | 16. \$ $7.60\frac{2}{3}$. | 24. \$ $402\frac{3}{10}$. |
| 8. \$ $9\frac{1}{3}$. | 17. \$ 537.12. | 25. \$ $312\frac{5}{12}$. |
| 9. \$ $2\frac{11}{12}$. | 18. \$ $263.40\frac{1}{2}$ | 26. \$ $573\frac{11}{18}$. |

## LESSON XLVI.—Pages 105, 106.

1. \$ 60 is $\frac{1}{2}$ of \$ 120, hence its interest will be $\frac{1}{2}$ as great; $\frac{1}{2}$ of \$ 9 is \$ $4\frac{1}{2}$. $\frac{1}{6}$ of \$ 9 = \$ $1\frac{1}{2}$. $\frac{1}{12}$ of \$ 9 = \$ $\frac{3}{4}$.

6. 6 years are 2 times 3 years, hence the interest for 6 years will be 2 times as much as for 3 years; 2 times \$ 20 = \$ 40.

Since the interest for 3 years is \$ 20, the interest for one year will be $\frac{1}{3}$ of \$ 20 = \$ $6\frac{2}{3}$, and the interest for 10 years will be 10 times \$ $6\frac{2}{3} =$ \$ $66\frac{2}{3}$. The interest for 4 years will be 4 times \$ $6\frac{2}{3} =$ \$ $26\frac{2}{3}$.

11. The interest at 3 per cent. is $\frac{1}{2}$ as great as at 6 per cent.; $\frac{1}{2}$ of \$ 27 = \$ $13\frac{1}{2}$. Since the interest at 6 per cent. is \$ 27, the interest at 1 per cent. will be $\frac{1}{6}$ of \$ 27 = \$ $4\frac{1}{2}$.

13. The interest of $ 100 for 1 year, at 1 per cent., is $ 1, and for 3 years is 3 times $ 1 = $ 3; since $ 3 is the interest at 1 per cent., $ 21 will be the interest at as many per cent. as $ 3 is contained times in $ 21, which are 7.

2. $ 6; 60 cts.; 6 cts.
3. $ 1.80; $ 18; $ 90.
4. $ 9; $ $4\frac{1}{2}$.
5. $ $11\frac{1}{2}$; $ 23.
7. $ $34\frac{1}{2}$; $ $80\frac{1}{2}$; $ $11\frac{1}{2}$.
8. $ 9; $ $\frac{2}{4}$; $ 3; $ $5\frac{1}{2}$.
9. 50 cts.; $16\frac{2}{3}$ cts.; $3\frac{1}{3}$ cts.
10. $ 5; $ $2\frac{1}{2}$; $ $1\frac{1}{2}$; $ $\frac{5}{8}$.
12. $ $2\frac{1}{7}$; $ $17\frac{1}{7}$.
14. 7 per cent.
15. $ 4; 5 per cent.
17. 3 per cent.
18. 2 per cent.
19. 7 per cent.

21. The interest of $ 50 for 1 year 3 months, at 1 per cent., is $ $\frac{5}{8}$; $ 3 ÷ $ $\frac{5}{8}$ = $4\frac{4}{5}$ = rate per cent.

22. The interest of $ 75 for 4 years 2 months, at 1 per cent., is $ $3\frac{1}{8}$; $ 12 ÷ $ $3\frac{1}{8}$ = $3\frac{21}{25}$ = rate per cent.

23. The interest of $ 48 for 1 year 4 months, at 1 per cent., is $ $1\frac{6}{25}$; $ 8 ÷ $ $1\frac{6}{25}$ = $12\frac{1}{2}$ = rate per cent.

24. The interest of $ 30 for 2 years 6 months, at 1 per cent., is $ $\frac{2}{4}$; $ 6 ÷ $ $\frac{2}{4}$ = 8 = rate per cent.

25. The interest of $ 100 for 2 years 2 months, at 1 per cent., is $ $2\frac{1}{6}$; $ 25 ÷ $ $2\frac{1}{6}$ = $11\frac{7}{13}$ = rate per cent.

26. The interest of $ 60 for 1 year 6 months, at 1 per cent., is $ $\frac{9}{10}$; $ 30 ÷ $ $\frac{9}{10}$ = $33\frac{1}{3}$ = rate per cent.

## LESSON XLVII.—Pages 107, 108.

In the first 11 examples we can use the interest of $ 1, or of $ 100, as is most convenient.

5. 2 years 4 months = $2\frac{1}{3}$ years. The interest of $ 100 for 1 year, at 4 per cent., is $ 4, and for $2\frac{1}{3}$ years it is $2\frac{1}{3}$ times $ 4 = $ $9\frac{1}{3}$; as $ 100 gains $ $9\frac{1}{3}$, it will require as many hundred dollars to gain $ 64, as $ $9\frac{1}{3}$ is contained times in $ 64, which are $6\frac{6}{7}$; hence $6\frac{6}{7}$ hundred dollars, or $ $685\frac{5}{7}$, is the principal required. Or,

*As in* 1 year it gains 4 per cent., in $2\frac{1}{3}$ years it will gain $2\frac{1}{3}$ *times* 4 *per cent.* = $9\frac{1}{3}$ per cent. = $\frac{28}{300}$ = $\frac{7}{75}$; as $\frac{7}{75}$ of *the principal* = $ 64, $\frac{1}{75}$ = $ $9\frac{1}{7}$, and $\frac{75}{75}$ = $ $685\frac{5}{7}$ = *the principal.*

| | | |
|---|---|---|
| 2. $100. | 9. $47$\frac{7}{10}$. | 16. 6 years. |
| 3. $50. | 10. $36$\frac{8}{14}$. | 17. 5 years. |
| 4. $33$\frac{1}{3}$. | 11. $360. | 18. 3 years. |
| 6. $62$\frac{1}{2}$. | 13. 10 years. | 19. 10 years. |
| 7. $333$\frac{1}{3}$. | 14. 10 years. | 20. 22$\frac{2}{9}$ years. |
| 8. $78$\frac{8}{12}$. | 15. 12$\frac{1}{2}$ years. | 21. 16$\frac{2}{3}$ years. |

22. 100 per cent. $\div$ 9 per cent. $= 11\frac{1}{9}$ = number of years at 9 per cent. $5\frac{5}{9}$ years at 18 per cent.

24. $\frac{2}{8}$ of $\frac{1}{8} = \frac{2}{15}$; $\frac{15}{15} \div \frac{2}{15} = 7\frac{1}{2}$ = number of years to double itself. $3\frac{3}{4}$ years to gain $\frac{1}{2}$ of itself. $1\frac{7}{8}$ years to gain $\frac{1}{4}$ of itself.

25. The interest of $1200 for 1 month, at 8 per cent., is $8; $80 $\div$ $8 = 10 = number of months.

26. The interest of $500 for 1 year, at 7 per cent., is $35; $350 $\div$ $35 = 10 = number of years.

## LESSON XLVIII.—Pages 109, 110.

3. 5 per cent. a year is 25 per cent. for 5 years, that is, $\frac{25}{100}$, or $\frac{1}{4}$, of the present worth; the present worth is $\frac{4}{4}$ of itself and $\frac{1}{4}$ added makes $\frac{5}{4}$, hence $\frac{5}{4}$ of the present worth = $50; if $\frac{5}{4} = \$50$, $\frac{1}{4} = \frac{1}{5}$ of $50, which is $10, and $\frac{4}{4} = 4$ times $10, which are $40 = present worth. Or,

$1 in 5 years, at 5 per cent., will amount to $\$\frac{5}{4}$; if $1 amount to $\$\frac{5}{4}$, it will take as many dollars to amount to $50, as $\$\frac{5}{4}$ is contained times in $50, which are 40.

15. We may find the present worth, as in the 3d example, and subtract from the given sum; or the discount may be found as follows:—

7 per cent. a year is 14 per cent. for 2 years, hence the discount is $\frac{14}{100} = \frac{7}{50}$ of the present worth; the present worth is $\frac{50}{50}$ of itself, and $\frac{7}{50}$ added make $\frac{57}{50}$, hence $\frac{57}{50}$ of the present worth = $100; if $\frac{57}{50} = \$100$, $\frac{1}{50} = \frac{1}{57}$ of $100, which is $1\frac{43}{57}$, and $\frac{7}{50} = 7$ times $1\frac{43}{57}$, which are $12\frac{43}{57}$ = the discount.

| | | |
|---|---|---|
| 4. $100. | 6. $5. | 8. $50. |
| 5. $100. | 7. $75. | 9. $60. |

| | | |
|---|---|---|
| 10. $ 20. | 13. $ 26. | 17. $ 7. |
| 11. $ 24. | 14. $ 24. | 18. $ 3. |
| 12. $ 27. | 16. $ 22$\frac{4}{5}$. | 19. $ 5$\frac{5}{6}$. |

## LESSON XLIX. — PAGES 110–113.

2. A $ 9 and B $ 6.

3. John $ 20 and Simon $ 15.

4. The elder $ 420 and the younger $ 240.

5. Edward 56 cts., Robert 28 cts., and John 14 cts.

6. $4 = \frac{8}{2}$, and $3\frac{1}{2} = \frac{7}{2}$, $\frac{8}{2} + \frac{7}{2} = \frac{15}{2}$; hence one has $\frac{8}{15}$, or 16 gallons, and the other $\frac{7}{15}$, or 14 gallons.

8. $\frac{4}{5} + \frac{5}{5} = \frac{9}{5}$ of the larger $= 27$; hence $\frac{1}{5}$ of the larger $= \frac{1}{9}$ of $27 = 3$, and $\frac{5}{5} = 5$ times $3 = 15 =$ the larger; $\frac{4}{5}$ of the larger $= 4$ times $3 = 12 =$ the smaller. Or,

As there are 9 fifths in both, one containing 4 of the 9 and the other 5, one must be $\frac{4}{9}$ of $27 = 12$, and the other $\frac{5}{9}$ of $27 = 15$.

9. I pay $\frac{1}{4}$ more than he, and he pays $\frac{1}{4}$ less than I; I pay $\frac{5}{9}$ of the whole, and he pays $\frac{4}{9}$; I ought to get $ 35 and he $ 28.

10. C pays $ 2 for each cow, hence D should put in as many cows as $ 2 is contained times in $ 10, which are 5.

11. 3 cows for 8 days is the same as 8 times 3 cows, or 24 cows, for 1 day, and 4 cows for 7 days is the same as 7 times 4 cows, or 28 cows, for 1 day; hence the whole is the same as $24 + 28 = 52$ cows for 1 day. $\frac{24}{52} = \frac{6}{13}$, and $\frac{28}{52} = \frac{7}{13}$; John should pay $\frac{6}{13}$ of $ 5 = $ $2\frac{4}{13}$, and Samuel $\frac{7}{13}$ of $ 5 = $ $2\frac{9}{13}$.

12. The 6 lazy men received 6 times as much as 1 lazy man, the 3 blind men received $3 \times 3 = 9$ times as much as 1 lazy man, and the 2 cripples received $2 \times 2 \times 3 = 12$ times as much as 1 lazy man; hence they all received $6 + 9 + 12 = 27$ times as much as 1 lazy man. If 27 times 1 lazy man's share $= 54$ cents, his share must be $\frac{1}{27}$ of 54 cents $=$ *2 cents; each cripple* received 6 times as much, or 12 *cents.*

## LESSON LVII.—Pages 126–130.

2. 23.
3. 48.
4. 27.
5. 35.
6. 36.
7. 48. [2 × 24]
8. 81. [3 × 27]
9. 23; 3; 69; 100; 169.
10. 225. [100 + 5 × 25]
11. 400; 41; 441.
13. 625. [400 + (25 — 20) × (25 + 20)]
729. [400 + 7 × 47]
14. 900; 961.
15. 1089; 1225; 1369.
16. 2704; 3025; 3364.
17. 3721; 4225; 5329.
18. 5625; 6561; 7569.
19. 10404; 12100; 13225.
20. 14400; 16129; 17689.
21. 19321; 21025; 22801.
22. 35721; 36481; 34969.
23. 46225; 47089; 52441.
24. 55225; 60025; 65025.
25. 85264; 110889; 133225.
26. 99225; 105625; 79524; 33856.
27. 64516; 68644; 78961.
28. 91809; 96721; 101761; 103684.
29. 126736; 127449; 105625.
31. $20\frac{1}{4}$; $12\frac{1}{4}$.
32. $56\frac{1}{4}$; $110\frac{1}{4}$.
33. $420\frac{1}{4}$; $930\frac{1}{4}$.
35. 96; 91; 84.
36. 10 and 3.
38. 399.
39. 899; 896; 891.
40. 1564; 1584; 1575.
43. $30956.
44. 18632; 20992; 16577.
45. 110889.
46. 23328 square inches.
48. 1402228; 1099745.

## LESSON LVIII.—Pages 130–132.

4. 10; 12; 15; 17; 18.
5. 11; 13; 14.
6. 12; 14; 11.
7. 11; 9; 22.
8. 21.
9. 18; 19; 23; 28.
10. 84; 63; 42; 36; 28.
11. 16 *qts.*; 37 qts.
12. 8 *cts.*; $2.08; 39 bot.
13. 42.
14. 72; 74; 63.
15. 51; 62; 82.
16. 92; 81; 71.
17. $16\frac{1}{3}$; $19\frac{1}{3}$; $20\frac{1}{3}$.
18. $24\frac{1}{7}$; 24; 21.
19. $24\frac{1}{7}$; $24\frac{4}{7}$; 25.
20. 18 barrels.
21. $108\frac{1}{2}$; 101; 106; 111.

22. 128.
23 and 24. 121; 131; 142.
25. $128\frac{4}{5}$; 285.
26. 181; 207.
27. $96\frac{1}{3}$.
28. $230\frac{3}{5}$; 193.
29. 111 hours.
30. $\$111\frac{1}{3}$.
31. $1\frac{1}{3}$; $11\frac{1}{3}$.
33. 1; 3; 2.
34. 178 pounds.
35. 234 slates.
36. 481 pks. 5 qts.; 794 pks. 1 quart.
37. 706 cloaks.

## LESSON LIX.—Pages 133–135.

2. 12; 13.
3. 12; 14; 15.
4. 17; 12; 15.
5. 16.
6. 15.
7. 8 libraries.
8. 24 companies.
9. 50 cases.
10. 82 yards.
11. 259.
12 and 13. 32.
14 and 15. 37.
16 and 17. 483.
18. 621.
19. 27; 23.
20. 27.
21. 23.
22. 28; 33; 38.
23. 28; 31; 35.
24. 91; 80; 96.
25. 37; 48; 32.
26. $27\frac{7}{11}$.
27. 31; 45; 64.
28. 69; 61; 81.
29. 52; 78.
30. 68; 79.
31. $34\frac{81}{84}$ h.; $34\frac{83}{84}$ hours.
32. 52 d.; $52\frac{4}{67}$ d.; *75* days.
33. 54; $65\frac{1}{2}$; $78\frac{1}{2}$.
34. 56 acres; $56\frac{14}{47}$ acres.

## LESSON LX.—Pages 135–138.

3. $2 \times 2 \times 3$; $2 \times 3$.
4. 1, 2, 3, 5, 7.
5. $2 \times 7$.
7. $3 \times 3 \times 7$.
8. $7 \times 7$; $7 \times 11$; $7 \times 13$.
9. 2. [23 and 29]
10. $5 \times 7$; $3 \times 13$.
11. 2. [31 and 37]
12. 2 times.
13. 2, 3, and 6 bushels;* 21, 14, and 7 bags.
14. 1484.

* *42 bags, holding* 1 bushel each, would also exactly contain the *in.*

15. 2205.
16. 2538.
17. $2418.
18. $4956; $6132; $1176.
19. 576; 2016; 4852.
20. 15367.
21. Prime; 11 × 13; 13 × 17; 17 × 19; 3 × 7 × 23.
22. 2 × 9 × 9; 8 × 41; 3 × 4 × 41; 3 × 4 × 47.
23. 25740; 17248; 213346.
28. 1818; 18180; 18[illegible]00.
31. $283.40.
32. 112000 pounds.
37. 62 acres.
38. 18 horses.
39. $90\frac{10}{13}$.
40. 66 lots.

## LESSON LXI.—Pages 138–140.

4. $\frac{9}{10}$.
5. $\frac{6}{10}$.
6. $\frac{75}{100}$.
7. $\frac{8}{10}$; $\frac{4}{10}$; $\frac{7}{10}$.
15. 0.05 £.; 0.15 £.; 0.35 £.
16. 7.5 s.; 0.375 £.
19. 14 cts.; $12\frac{1}{2}$ cts.; 11 cts.
20. $4.29.
21. 0.06 lb.; 0.19 lb.: 0.25 lb.
22. 0.50 ft.; 0.62 ft.; 0.69 ft.
23. 0.48 m.; 0.403 m.
24. 0.465 inch; 0.549 inch; 0.604 inch.
25. 0.531 A.; 0.609 A.
26. 0.66 C.; 0.6 C.; 0.34C.
27. 0.92 d.; 0.7 d.; 0.64 d.
28. 0.75 lb.; 0.5 lb.; 0.656 lb.

## LESSON LXII.—Pages 141–143.

1. 1.84.
2. $3.26.
3. $50.
4. 1.269.
5. $5.76.
6. 3.56; 2.61; 5; 4.49.
7. 2.689.
8. $12.74.
9. 5.13.
10. 0.897.
11. 0.003.
12. 12.625.
13. 1.02.
14. 0.192;* 40.5.
15. 13.97; 18.8; 29.43.
16. 6.92; 200.2.
17. $218.40.
18. 0.01; 0.001; 0.0001.

* Multiply like common fractions. $\frac{1}{1000}$ of 24 is $\frac{24}{1000}$, and $\frac{8}{1000}$ is 8 times $\frac{24}{1000} = \frac{192}{1000} = 0.192$.

21. $8.10.
22. and 24. $9.59; $57.54; $115.08.
25. $4.50; 45 cts.
26. $7.92.
27. $15.20.
28. $46.63.
29. $25.44; $76.32.
30. $15.15; $60.59.
31. $66; $132.
32. $0.20; $0.50.
34. $5.66; $45.53.
35. $178.94.
36. $55.56.
37. $14.16.
38. $20.51.

## LESSON LXIII.—Pages 143–145.

1. 1.28; 1.22; 1.225.
2. 0.65; 0.425; 0.45.
3. 30; 300; 3000.
4. 1.775; 4.216; 4.40.
5. 4.033.
6 and 7. 16; 160.
8. 9.1; 0.91.
9. 0.167; 0.143; 0.125; 0.111.
10. 1.6.
11. 0.3333.
12. 13; 1.3; 13.
13. 15.17.
14. $23.46.
15. 0.333; 0.167.
16. $1.43; $1.429.
17. $1.111.
18. 0.091.
19. 0.08.
20. 200; 3000; 12.5; 125.
21. 16 pounds.
22. $60.
23. 6.7 yards.
24. 5.9; 590; 0.59.
25. 2.61; 26.14; 261.4.
26. 5.29 hours.
27. 12 bushels.
28. 10.5 bushels.

## LESSON LXIV.—Pages 145–147.

1. $5.20. 2. $210.29. 3. $168.

4. 33 is 3 times 11; $3 \times \$0.54 = \$1.62 =$ sum required.

5. $3 + 4 = 7$, $2 + 3\frac{1}{2} = 5\frac{1}{2}$; if 7 were 12, 1 would be $\frac{1}{7}$ of $12 = \frac{12}{7}$, and $5\frac{1}{2}$ would be $5\frac{1}{2} \times \frac{12}{7} = 9$.

6 and 7. $207 - 29 = 178 =$ the less. Subtract the difference from the greater.

*8 and 9.* $174 + 28 = 202 =$ the greater. Add the *difference* to the less.

*10.* $\frac{1}{2}$ *of* 232 lbs. = 116 lbs. The gain on one half was

9. The chain cost $\frac{1}{8}$ as much as the watch, and the key cost $\frac{5}{100} = \frac{1}{20}$ as much as the watch; hence they all cost $\frac{1}{8} + \frac{1}{8} + \frac{1}{20} = \frac{5}{4}$ as much as the watch; if $\frac{5}{4} = \$125$, $\frac{1}{4} = \$100$ = cost of the watch; $\frac{1}{5}$ of $\$100 = \$20$ = cost of the chain, and $\frac{1}{20}$ of $\$100 = \$5$ = cost of the key.

10. 100 per cent. $\div 5\frac{2}{3}$ per cent. $= 18\frac{3}{17} =$ no. years.

11. $\frac{1}{8} + \frac{1}{8} = \frac{8}{15}$; $\frac{15}{15} \div \frac{8}{15} = \frac{15}{8} = 1\frac{7}{8} =$ no. days.

12. 20 per cent. [Analyzed like example 31, Lesson XLII.]

13. \$ 60, at 7 per cent., gains \$ 4.20 in 1 year; hence it would take as many years to gain \$ 6.30 as \$ 4.20 is contained times in \$ 6.30, which gives $1\frac{1}{2}$ years = 1 year 6 months.

14. 10 gallons are worth 10 times \$ 8, which are \$ 80; at \$ 7 a gallon, it will take as many gallons to amount to \$ 80, as \$ 7 is contained times in \$ 80, which are $11\frac{3}{7}$; $11\frac{3}{7} - 10 = 1\frac{3}{7} =$ number of gallons of water.

15. If they start from the same point, B must gain 20 miles to overtake A; as he gains 2 miles an hour, it will take him 10 hours to gain 20 miles.

16. \$ 22.77.

17. \$ 100 is $\frac{100}{250}$, or $\frac{2}{5}$, of \$ 250; hence \$ 250 must be loaned $\frac{2}{5}$ of a year to requite the favor of \$ 100 for 1 year.

18. If in $\frac{25}{2}$ of a year it gain 100 per cent., in 1 year it will gain $\frac{2}{25}$ of 100 per cent., which are 8 per cent.

19. If the rest were sold at \$ 2.50 apiece, only $\frac{5}{4}$ of the cost of the lot would be received; hence $\$2\frac{1}{2} = \frac{5}{4}$ of the required price, and $\frac{1}{4} = \$3\frac{1}{8} =$ the required price.

20. $\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$, $\frac{1}{4}$ of $\frac{2}{8} = \frac{1}{8} =$ son's share; $\frac{2}{3} - \frac{1}{6} = \frac{1}{2}$, $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3} =$ daughter's share; $\frac{1}{8} + \frac{1}{6} + \frac{1}{3} = \frac{9}{14} = \$900$; $\frac{14}{14} = \$1400 =$ whole estate.

21. $\frac{4}{5} - \frac{1}{4} = \frac{11}{20}$, $\frac{5}{5} - \frac{11}{20} = \frac{9}{20}$, $\frac{9}{20}$ of $\frac{4}{5} = \frac{9}{25} =$ last remainder = 27 peaches; $\frac{20}{20} = 60$ peaches = whole number.

22. A, B, and C pay $\frac{9}{10}$ of the rent, hence $6 + 8 + 4 = 18$ must be $\frac{9}{10}$ of the whole number of cows; $\frac{1}{10} = 2$ cows = number D put in, and $\frac{10}{10} = 20$ cows = whole number. *A paid* $\frac{6}{20}$, or $\frac{3}{10}$, of \$ 16; B paid $\frac{8}{20}$, or $\frac{4}{10}$, of \$ 16; and *C paid* $\frac{4}{20}$, or $\frac{2}{10}$, of \$ 16.

23. It was sold for 75 per cent. of the cost, and the difference between its proper value and the price received was + 30 = 55 per cent. of the cost; hence this difference is $\frac{55}{75}$, or $\frac{11}{15}$, of the price received; $\frac{11}{15}$ of $35 = $25\frac{2}{3}$. Or,

Since 75 per cent. = $35, 1 per cent. = $\frac{1}{75}$ of $35 = $\frac{7}{15}$, and 55 per cent. = 55 times $\frac{7}{15} = \frac{77}{3} = $25\frac{2}{3}$.

24. It was sold at 80 per cent., and, to gain 20 per cent. it should have been sold at 120 per cent.; hence it should have been sold at $\frac{120}{80}$, or $\frac{3}{2}$, of the price received; $\frac{3}{2}$ of $30 = $45. Or,

Since 80 per cent. = $30, 1 per cent. = $\frac{1}{80}$ of $30 = $\frac{3}{8}$, and 120 per cent. = 120 times $\frac{3}{8} = $45.

25. 8 horses will consume 2 times as much as 4 horses, and they will consume 2 times as much in 12 days as in 6 days; 2 × 21 bushels = 42 bushels, 2 × 42 bushels = 84 bushels.

26. $\frac{22}{7}$ of 3 rods = $9\frac{3}{7}$ rods = its circumference.

27. $\frac{7}{22}$ of 11 miles = $3\frac{1}{2}$ miles = its diameter.

28. Twice their sum must be 2 times 19, or 38; the difference between twice the sum, and twice the first and 5 times the second, must be 3 times the second; the difference between 38 and 74 is 36; hence 3 times the second must be 36, and the second must be $\frac{1}{3}$ of 36 = 12. The first is 19 — 12 = 7.

29. Williams puts in $\frac{4}{3}$ as much as Brown, hence the whole is $\frac{4}{3} + \frac{3}{3} = \frac{7}{3}$ of Brown's share; as $\frac{7}{3} = $700, $\frac{3}{3} = $300 = Brown's share, and $\frac{4}{3} = $400 = Williams's share.

30. A cord of wood costs $17 — the cost of a barrel of flour, $10 more than the cost of a cord of wood = $27 — the cost of a barrel of flour = the cost of 2 barrels of flour; hence $27 = the cost of 3 barrels of flour, and $9 = the cost of 1 barrel of flour. $17 — $9 = $8 = the cost of 1 cord of wood.

31. 1 of the hound's leaps = $1\frac{1}{2}$ of the hare's; while the hound is taking 1 leap, the hare takes $1\frac{1}{2}$ leaps; $1\frac{1}{2} - 1\frac{1}{3} = \frac{1}{6}$, hence, while the hound is taking 1 leap, he gains $\frac{1}{6}$ of a hare's leap, and he must take as many leaps to gain 25 hare's leaps, as $\frac{1}{6}$ of a leap is contained times in 25 leaps, which are 150. Ans. 150 leaps.

32. If 20 feet is the shadow of 30 feet, 1 foot will be the shadow of $1\frac{1}{2}$ feet, and 25 feet will be the shadow of 25 times $1\frac{1}{2}$ feet $= 37\frac{1}{2}$ feet. Or,

The shadow is $\frac{2}{3}$ as long as the pole; 25 feet are $\frac{2}{3}$ of $37\frac{1}{2}$ ft.

33. As A's $= \frac{1}{2}$ of B's $+ 15$, A's and B's $= 1\frac{1}{2}$ times B's $+ 15 = 54$; hence $\frac{2}{3}$ of B's $= 39$, and B's $= \frac{3}{2}$ of $39 = 26$ sheep. A's $= 54 - 26 = 28$ sheep.

34. Smith puts in 3 parts, and Robinson 4 parts; Smith's is $\frac{3}{7}$ and Robinson's $\frac{4}{7}$ of the capital.

35. The 2d no. $= 24 -$ the 1st no., and twice the 2d $= 48 -$ twice the 1st; 3 times the 1st $=$ twice the 2d $+ 17$; hence 3 times the 1st $= 65 -$ twice the 1st, 5 times the 1st $= 65$, and the first $= 13$. The second no. $= 24 - 13 = 11$.

36. $\frac{7}{22}$ of 22 in. $= 7$ in. $=$ diam.; $\frac{7}{2} \times 11 = \frac{77}{2} = 38\frac{1}{2}$ square inches $=$ area.

37. Each side of the square is $\frac{1}{4}$ of 22 inches $= 5\frac{1}{2}$ inches, and its area is $\frac{11}{2} \times \frac{11}{2} = \frac{121}{4} = 30\frac{1}{4}$ square inches; $38\frac{1}{2} - 30\frac{1}{4} = 8\frac{1}{4}$ square inches.

38. They cost 24 cents; half of them sold for 15 cents and half for 10 cents, hence the whole sold for 25 cents; 25 cents $- 24$ cents $= 1$ cent $=$ the gain.

39. The time past midnight was $\frac{4}{3}$ of itself; $\frac{4}{3} - \frac{1}{3} = \frac{3}{3} =$ time from midnight to noon $= 12$ hours, $\frac{1}{3} = \frac{1}{3}$ of 12 hours $= 4$ hours $=$ the time past noon. Ans. 4 P. M.

40. $\frac{2}{5} - \frac{1}{5} = \frac{1}{5}$; $\frac{1}{5}$ of the time past midnight $= 12$ hours, $\frac{1}{5} = \frac{12}{5}$ h., $\frac{4}{5} = \frac{48}{5}$ h. $= 9\frac{3}{5}$ h. $= 9$ h. 36 m. past noon.

41. As $\frac{1}{3}$ of the time to midnight $= \frac{1}{3}$ of the time past noon, $\frac{6}{6}$, or the whole time to midnight $= 6 \times \frac{1}{3} = \frac{6}{3} = 2$ times the time past noon; hence the time from noon to midnight, or 12 hours, was 3 times the time past noon, and the time was $\frac{1}{3}$ of 12 hours $= 4$ hours past noon.

42. The 4 sides of the square measure 20 miles; A gains $1\frac{1}{2}$ miles an hour on B, and it will take him as many hours to overtake B as $1\frac{1}{2}$ miles is contained times in 20 miles, which are $13\frac{1}{3}$. B will have travelled $13\frac{1}{3} \times 3$ miles $= 40$ miles $=$ 2 times around the square. A will have travelled $13\frac{1}{3} \times 4\frac{1}{2}$ miles $= 60$ miles $= 3$ times around the square.

43. After the first change there are 25 gal. of each; $\frac{1}{5}$ of 25 gal. $= 5$ gal. of wine are then drawn off, and there are $25 - 5 = 20$ gal. of wine in the cask; $50 - 20 = 30$ gal. of water. Or,

$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$; $\frac{4}{5}$ of $\frac{1}{2} = \frac{2}{5}$; $\frac{2}{5}$ of $50 = 20$ = no. gal. of wine; $50 - 20 = 30$ = no. gal. of water.

44. At first $\frac{1}{3}$ of the wine is drawn off and $\frac{2}{3}$ remain, then $\frac{1}{3}$ of $\frac{2}{3}$ is drawn off and $\frac{5}{6}$ of $\frac{2}{3} = \frac{5}{9}$ of the wine remain; $\frac{5}{9}$ of 50 gal. $= 27\frac{7}{9}$ gal. = quantity of wine remaining. As the quantities added are the same as those drawn off, there are 50 gal. of the mixture; $50 - 27\frac{7}{9} = 22\frac{2}{9}$ = no. gal. water.

It will be seen that the 43d and 44th bear the same relation to each other as the 48th and 49th.

45. The wolf can eat $\frac{2}{5}$ of a sheep in 1 day, and $\frac{1}{2}$ of $\frac{2}{5} = \frac{1}{5}$ in $\frac{1}{2}$ of a day; the hound can eat $\frac{4}{13}$ of a sheep in 1 day, and $\frac{1}{4}$ of $\frac{4}{13} = \frac{1}{13}$ in $\frac{1}{4}$ of a day; $\frac{1}{5} + \frac{1}{13} = \frac{18}{65}$, $\frac{65}{65} - \frac{18}{65} = \frac{47}{65}$ = the part of the sheep left to be eaten. $\frac{4}{13} + \frac{1}{4} = \frac{29}{52}$ = what the hound and mastiff together can eat in 1 day; it will take them as many days to eat $\frac{47}{65}$ of a sheep as $\frac{29}{52}$ is contained times in $\frac{47}{65}$, which are $1\frac{148}{145}$. Ans. $1\frac{148}{145}$ days.

46. A paid 75 per cent. of his value and gained 50 per cent. of his value, or $\frac{50}{75} = \frac{2}{3} = 66\frac{2}{3}$ per cent. of the cost. *B* would lose 50 per cent. of his value on 125 per cent. of his value, or $\frac{50}{125} = \frac{2}{5} = 40$ per cent. of what he paid.

47. By the last condition, the weight of the 2d cup $= 3$ times the weight of the 1st cup $- 10$ oz., hence twice the 2d $= 6$ times the 1st $- 20$ oz.; by the other condition, twice the 2d = the 1st $+ 10$ oz.; therefore 6 times the 1st $- 20$ oz. = the 1st $+ 10$ oz., 5 times the 1st $- 20$ oz. $= 10$ oz., 5 times the 1st $= 30$ oz., and the 1st cup weighs $\frac{1}{5}$ of 30 oz. $= 6$ oz. The 2d weighs $3 \times 6$ oz. $- 10$ oz. $= 8$ oz.

48. First, $\frac{1}{3}$ of the wheat would be taken away, and $\frac{2}{3}$ left. After each change, there would still be 30 bushels in the bin; hence $\frac{1}{3}$ of the mixture, and consequently of the remaining quantity of wheat, would also be taken away the last two times, and $\frac{2}{3}$ would be left. After the first change, $\frac{2}{3}$ of 30 bushels of wheat would remain; after the second change, $\frac{2}{3}$ of $\frac{2}{3} = \frac{4}{9}$

would remain; and after the third change, $\frac{2}{3}$ of $\frac{4}{9} = \frac{8}{27}$ of 30 bushels $= 8\frac{8}{9}$ bushels of wheat would remain. $30 - 8\frac{8}{9} = 21\frac{1}{9}$ = number of bushels of rye.

49. 10 bushels of the mixture would each time be taken from 40 bushels, hence $\frac{1}{4}$ of the wheat would be taken, and $\frac{3}{4}$ would remain; $\frac{3}{4}$ of $\frac{3}{4} = \frac{9}{16}$, $\frac{3}{4}$ of $\frac{9}{16} = \frac{27}{64}$, $\frac{27}{64}$ of $30 = 12\frac{24}{32}$ = no. bush. of wheat. $30 - 12\frac{24}{32} = 17\frac{1}{32}$ = no. bush. rye.

50. At the end of 4 months, the sum paid and the interest amount to \$ 540; 120 per cent. $= \frac{6}{5}$, $\frac{6}{5}$ of \$ 500 = \$ 600; the present worth of \$ 600 due 3 months hence = \$ $566\frac{2}{53}$; \$ $566\frac{2}{53}$ − \$ 540 = \$ $26\frac{2}{53}$ = the gain.

51. At the earliest time mentioned, the father's age was 3 times his son's; then 3 times the son's age + 12 yrs. = twice the son's age at the latest time mentioned, or twice the son's age + 24 yrs.; 3 times the son's age = 2 times the son's age + 12 yrs., the son's age = 12 yrs. and the father's age = 3 × 12 yrs. = 36 yrs. At the required time, both were 4 years older; the son was 16 years old, and the father was 40 years old.

52. 5 per cent. of $\frac{2}{3} = \frac{1}{120}$, and 6 per cent. of $\frac{5}{8} = \frac{6}{160}$; $\frac{1}{120} + \frac{6}{160} = \frac{9}{160}$; if $\frac{9}{160}$ of his money = \$ 90, $\frac{1}{160}$ = \$ 10, and $\frac{160}{160}$ = \$ 1600; $\frac{5}{8}$ of \$ 1600 = \$ 600 = sum let at 5 per cent.; \$ 1600 − \$ 600 = \$ 1000 = sum let at 6 per ct.

53. $\frac{2}{5} - \frac{1}{8} = \frac{1}{15}$; if $\frac{1}{15}$ of the money = \$ $\frac{5}{8}$, $\frac{15}{15}$ = \$ 25.

54. On every pound at 40 cts. there is gained 4 cts., and on every pound at 50 cts. there is lost 6 cents; hence, to make the gain and loss equal, there must be 6 lbs. at 40 cts. to every 4 lbs. at 50 cts., or the 40 lbs. must be divided into 2 parts which shall be to each other as 3 to 2. $\frac{3}{5}$ of 40 = 24 = no. lbs. at 40 cts., $\frac{2}{5}$ of 40 = 16 = no. lbs. at 50 cents.

55. As the 1st + the 2d = the 3d, the sum of the three = twice the 3d = 18; hence the 3d = $\frac{1}{2}$ of 18 = 9. From the last condition, the 1st + the 3d = twice the 2d, and the sum of the three = 3 times the 2d = 18; hence the 2d = $\frac{1}{3}$ of 18 = 6. 18 − 6 − 9 = 3 = the 1st.

56. 6 men and 6 boys would receive \$ 10; 2 men would receive \$ 10 − \$ 8 = \$ 2, and 1 man would receive $\frac{1}{2}$ of \$ 2 =

$ 1 per day. 3 men would have 3 dollars; 3 boys would h
$ 5 — $ 3 = $ 2, and 1 boy would receive $\frac{1}{3}$ of $ 2 = $
per day.

57. Twice the first sum = the second sum, that is, tw
the no. + 6 = the no. + 14, and the no. = 14 — 6 = 8

58. As B and C have 11 times as much as A, they all h
12 times A's = 12 × $ 3 = $ 36; as A and C have 3 tin
as much as B, they all have 4 times B's; hence 4 times
= $ 36, and B's money = $\frac{1}{4}$ of $ 36 = $ 9. As B and
have 11 × $ 3 = $ 33, C has $ 33 — $ 9 = $ 24.

59. If 5 per cent. = $ 90, 100 per cent. = $ 1800 =
whole sum at interest. $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{4} = \frac{3}{12}$, and $\frac{1}{6} = \frac{2}{12}$; her
$ 1800 is to be divided into 3 parts which shall be to ea
other as 4, 3, and 2. Smith's is $\frac{4}{9}$ of $ 1800 = $ 800, Jone
is $\frac{3}{9}$ of $ 1800 = $ 600, and Brown's is $\frac{2}{9}$ of $ 1800 = $ 400

60. A and B pay $\frac{1}{2}$ of the $ 76 * without C's aid. 4 hor
eat as much as 4 × 4 sheep = 16 sheep, and 8 cows eat
much as 8 × 3 sheep = 24 sheep; hence A's horses and co
eat as much as 40 sheep. B's 6 horses and 12 sheep ea
much as 36 sheep, and the whole eat as much as 76 she
A must pay $\frac{40}{76}$, or $\frac{10}{19}$, of $ 38 = $ 20, and B must pay $\frac{9}{19}$, of $ 38 = $ 18. After C put in 24 sheep, the whol
as much as 100 sheep; hence A must pay $\frac{40}{100}$, or $\frac{2}{5}$, of
= $ 15$\frac{1}{5}$, B must pay $\frac{36}{100}$, or $\frac{9}{25}$, of $ 38 = $ 13$\frac{17}{25}$, a
must pay $\frac{24}{100}$, or $\frac{6}{25}$, of $ 38 = $ 9$\frac{8}{25}$. A must pay
$ 20 + $ 15$\frac{1}{5}$ = $ 35$\frac{1}{5}$; B must pay $ 18 + $ 13
$ 31$\frac{17}{25}$; C must pay $ 9$\frac{8}{25}$.

61. Robinson's labor is worth half as much, and Harr
$\frac{3}{5}$ as much, as if they worked all the time; hence 200 m
divided in the proportion of $\frac{1}{2}$, 4, and 2, or 5, 8, and 4.
inson should receive $\frac{5}{17}$ of $ 200 = $ 58$\frac{14}{17}$, Savag
$ 200 = $ 94$\frac{2}{17}$, and Harrison $\frac{4}{17}$ of $ 200 = $ 47$\frac{1}{17}$

* In the earlier editions of the Arithmetic this is erroneo
ed $ 77.

THE END.